The Effect of "Invisible" Tax Preferences on Investment and Tax Preference Measures

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Abstract: This paper develops and analyzes a model in which tax considerations and financial reporting considerations have countervailing effects on a firm's investments in internally developed intangible assets. It also proposes and estimates a new measure of tax preferences, which we call the economic effective tax rate. This measure reflects both investments in intangible assets and the use of debt financing, neither of which generates a book-tax difference. Our measure indicates that the economic effective tax rate was about 16 percent between 1988 and 2005, when the statutory tax rate was either 34 or 35 percent.

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1. Introduction

We investigate two consequences of the immediate expensing of investments in internally developed intangible assets for both tax and financial reporting purposes. First, we present a model in which both the tax advantages and financial reporting disadvantages of immediate expensing affect firm investment decisions. We show that the optimal ratio of investment in intangible assets to tangible assets could be greater than or less than the ratio from a benchmark case in which intangible assets have neither tax advantages nor financial reporting disadvantages. This paper is similar in spirit to Shackelford, Slemrod, and Sallee (2006) in that both accounting and taxable income are important to firm managers. Our setting allows us to explore the trade-off between tax incentives and financial reporting considerations through an economic model of firm investment in which financial reporting considerations enter the manager's objective function endogenously via the effect of accounting disclosures on stock price. The economic model we develop illustrates how tax and financial accounting reporting rules have countervailing effects on corporate investment in internally developed intangible assets.

Second, we propose a measure of the extent to which a firm's activities are tax-favored due to investments in intangible assets and the use of debt financing. Investments in intangible assets are expensed instead of capitalized and depreciated, while the use of debt financing yields tax-deductible interest expense payments instead of non-deductible dividend payments. Both investments in intangible assets and interest payments are expensed for both tax and financial reporting purposes and therefore do not generate book-tax differences. Accordingly, these fundamental investment and financing choices do not reduce the firm's accounting effective tax rate, but they do reduce the government's claim on the firm's pre-tax cash flows. Our measure, which we call the economic effective tax rate, reflects the effects of investments in intangible assets and uses of debt financing on a firm's tax burden, which traditional financial accounting measures fail to capture.

The government's claim on the firm's pre-tax cash flows is lower relative to the alternatives of capitalization and depreciation of intangible investments and non-deductibility of dividend payments to providers of equity capital. Accordingly, we estimate that firms faced an average economic effective tax rate of about 16 percent between 1988 and 2005, during which time the statutory rate was either 34 or 35 percent. On average, about two-thirds of the difference between our measure and the statutory tax rate is attributable to intangible assets and about one-third is attributable to the use of debt financing. Both the effect of intangible assets and the use of debt financing on our measure vary across industries. The effect of intangible assets increased between 1988 and 2005, whereas the effect of debt financing remained stable over that time period.

Both issues that we explore arise because external users of accounting information are unable to distinguish investments in internally developed intangible assets from operating expenses. The inability of investors to distinguish investments in intangible assets from operating expenses gives the firm manager an incentive to decrease such investments in an effort to boost stock price. The inability of policy makers to distinguish investments in intangible assets from operating expenses makes it difficult to evaluate the extent to which firms benefit from "invisible" tax preferences that do not create book-tax differences. Finally, the corporate finance literature indicates that investment in tangible, as opposed to intangible assets, supports more debt.[1] Consistent with this literature, we find that the effects of intangible assets and the effects of debt financing on our tax preference measure are negatively correlated, resulting in less intra-industry variation when both tax preferences are considered simultaneously.

In section 2 we discuss previous research on intangible assets and tax preference measures. In section 3 we extend the model developed by Kanodia, Sapra, and Venugopalan (2004) to show how the tax and financial reporting treatments of investments in intangible assets have countervailing effects on a firm's investments in intangible assets relative to tangible assets. In section 4 we derive a measure of tax preferences by modeling the firm as a collection of projects that feature both tangible and intangible investments that are in part financed with tax-favored debt. In section 5 we estimate our measure for U.S. firms between 1988 and 2005. Section 6 concludes. All proofs are in the appendix.

2. Previous research

2.1 Investments in internally developed intangible assets

Internally developed intangible assets are an important and growing source of firm value. These investments include the development of intellectual property (including, but not limited to, R&D expenditures), the creation of brand value via advertising, and the generation of firm-specific human capital. The rise in importance of these assets strains existing systems of financial and tax accounting, which evolved in an era in which tangible and financial assets were the dominant sources of firm value. This

[1] Myers (1977) develops a model that predicts the debt-to-value ratio will be lower the larger the proportion of firm value represented by intangible, rather than tangible assets, while Smith and Watts (1992) provided empirical support to Myers' claim.

strain arises because in general, firms expense investments in internally developed intangible assets for both tax and financial reporting purposes. Blair and Wallman (2001) provides an overview of the economic, accounting and taxation issues associated with intangible assets.

Our interpretation of intangible assets is an expansive one. While intangible assets include legally protected intellectual property such as patents, it is not limited to what would be regarded as property in a legal sense. It includes firm-specific human capital of the workforce and organizational capital such as Wal-Mart's computerized supply chain. An intangible asset is a claim to future benefits that does not have a physical or financial embodiment (Lev 2001, 5). At the broadest level, we distinguish investments in intangible assets from operating expenses, which consist of direct labor, direct materials, manufacturing overhead, marketing and administrative expenses that support current-period revenues with no direct effect of future cash flows (Kanodia et al. 2004, 96).

The literatures in public finance economics and financial accounting frame investments in intangible assets in very different ways. Public finance economists note that the ability to expense investments in intangible assets when made provides them with an effective tax rate of zero, which represents a substantial departure from tax neutrality (Gravelle 1994, 209). Therefore, public finance economists generally claim that a zero effective tax rate may induce excessive private investment in intangible assets.[2]

More recent corporate tax policy debates surrounding tax incentives and corporate investment have focused on this increasing importance of intangible assets, and the fact that these assets are already being expensed under current law. Both Neubig (2006) and

[2] This excessive investment could be socially beneficial, however, to the extent that intangible investments generate positive externalities because the firm cannot capture all of the benefits these investments create.

Sullivan (2006) point out that tax incentives that offer immediate expensing or accelerated depreciation of investments in tangible property are less desirable than tax rate reductions because so many investments in internally developed intangible assets are already expensed under current law.

The financial accounting literature, in contrast, largely reflects a view that accounting for investments in intangible assets as expenses rather than assets induces underinvestment. Lev (2001, 95) argues that the inability of investors to distinguish operating expenses from investments in intangible assets creates an information asymmetry that raises the cost of capital from such investments, thereby deterring them. Kanodia and Mukherji (1996) and Kanodia et al. (2004) develop this idea in formal models of firm investment decisions when capital market participants cannot distinguish operating expenses from investments in internally developed intangible assets. These models show that the nonobservability of intangible investments creates an incentive for the firm to cut back on these investments relative to a world where investors can disentangle investments in intangible assets from operating expenses.

2.2 Tax preference measures

The accounting literature on the measurement of tax preferences has focused on the accounting effective tax rate, the ratio of tax expense to pre-tax book income (Gupta and Newberry 1992; Shevlin and Porter 1992), or on related measures driven by book-tax differences (Wilkie 1992; Wilkie and Limberg 1993).

Fullerton and Lyon (1988) show that effective tax rate measures that exclude the taxation of intangible capital are misleading. In an analysis restricted to investments in R&D and advertising, they estimate the effect of expensing these investments for the

1983 tax year. They estimate that about 11 percent of the assets in their sample were associated with intangible assets.

Consistent with our expansive view of intangible assets, we measure the value of a firm's intangible assets indirectly, by taking the difference between the market value of the claims to the firm's assets and the book value of the firm's assets. Using our approach, we estimate that firms in our sample invest about 51 cents in internally developed intangible assets for every dollar invested in tangible assets.

Our study is similar in spirit to Dunbar and Sansing (2002), who examine the ratio of tax expense to the pre-tax market return on equity. Our results suggest an average economic effective tax rate, ignoring the effects of debt financing, of about 23 percent between 1992 and 1996, while the Dunbar and Sansing (2002) explicit tax rate measure is 26 percent over the same time period.

3. Economic model of the tension between tax and financial reporting

3.1 Basic model

A firm has access to a production technology that transforms current investments in tangible assets (*K*) and intangible assets (*N*) into output (*q*) in the following fashion:

$$f(K,N) = q. \tag{1}$$

The firm chooses its investments on date zero. The production technology generates output *q* on date one and on each succeeding date as long as both assets remain productive. Each unit of output generates an expected operating cash flow of $\mu > 0$. The date one operating cash flow, denoted *x*, is drawn from a continuous probability distribution with strictly positive support on the interval $[\underline{x}, \infty)$, and has a mean of μq and a variance of σ^2. The lower bound $\underline{x}$ is sufficiently high that the firm is better off if it operates the project instead of liquidating it. On each date j after the cash flow is received, the productivity of the intangible assets drops to zero with probability λ and the project is abandoned. With probability 1−λ, the project continues to generate positive operating cash flows in the future, where the expected cash flow on date $j+1$ is equal to the realized cash flow on date j. When the project is abandoned, the tangible assets survive with no reduction in value and are either sold at their historical cost K or redeployed within the firm in a new project. The intangible assets generate no further value to the firm once the project is abandoned.

The firm operates in a single taxing jurisdiction. Operating cash flows are taxed at a rate of τ. The tangible investment K is neither expensed nor depreciated for tax purposes because the tangible assets do not decrease in value over time. The intangible investment N is subject to one of two tax treatments. Under the *capitalization regime*, the full amount of the investment N is capitalized and subsequently deducted in computing taxable income under Internal Revenue Code §165 when the project is abandoned. Under the *expensing regime*, the full amount of the investment N is deducted in computing taxable income on date zero.

The firm chooses investments K and N to maximize the present value of the after-tax cash flows that the project generates, discounted at the project's after-tax weighted average cost of capital ρ, less the after-tax cost of the investments. Given investments K and N, the present value of the project's expected after-tax cash flows, not including the after-tax cost of the investments, is

$$\sum_{j=1}^{\infty} \frac{[(1-\tau)\mu q+\lambda K](1-\lambda)^{j-1}}{(1+\rho)^{j}} = \frac{(1-\tau)\mu q+\lambda K}{\rho+\lambda}. \tag{2}$$

Under the expensing regime, the firm's date zero maximization problem is

$$\max_{K,N}\left\{\frac{(1-\tau)\mu f(K,N)+\lambda K}{\rho+\lambda}-K-N(1-\tau)\right\}. \tag{3}$$

Under the capitalization regime, the firm's date zero maximization problem is

$$\max_{K,N}\left\{\frac{(1-\tau)\mu f(K,N)+\lambda(K+\tau N)}{\rho+\lambda}-K-N\right\}. \tag{4}$$

Using (3) and (4), we can see that the expensing regime is more favorable than the capitalization regime. We show this by comparing the after-tax cost of the intangible investment under the expensing regime, $N(1-\tau)$, to the after-tax cost of the intangible investment under the capitalization regime, $N\left(1-\frac{\tau\lambda}{\rho+\lambda}\right)$.

3.2 Investment in a full information environment

Next, we consider the trade-off between tax considerations that induce private investment in intangible assets, and financial reporting considerations that deter these investments. First, we develop a benchmark case in which investors can disentangle investments in intangible assets from operating expenses, and therefore, the financial reporting disincentive to invest in intangibles is absent. We refer to this as the *full information environment*. The firm chooses investments *K* and *N* to maximize the present value of the after-tax cash flows that the project generates, discounted at the project's after-tax weighted average cost of capital ρ, less the after-tax cost of the investments. We focus on the ratio of intangible investments to tangible investments $\frac{N}{K}$ under each tax regime because this provides a natural way to see how tax and financial reporting considerations affect the composition of the firm's capital stock. To do this, we impose additional structure on the problem by assuming that the firm has access to a Cobb-Douglas production technology of the form

$$f(K,N) = K^{\alpha} N^{\beta} \tag{5}$$

with the productivity parameters $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$. This functional form is consistent with the idea that investments in intangible assets and investments in tangible asset are made to support each other.

Proposition 1 presents the investment ratio $\frac{N^*}{K^*}$ associated with the solutions to the firm's maximization problem under the expensing regime characterized in (3) and the capitalization regime characterized in (4).

Proposition 1: In the full information environment:

(a) under the expensing regime, $\frac{N^*}{K^*} = \frac{\beta\rho}{\alpha(1-\tau)(\rho+\lambda)}$; and

(b) under the capitalization regime, $\frac{N^*}{K^*} = \frac{\beta\rho}{\alpha[\rho+\lambda(1-\tau)]}$.

A comparison of the investment ratios in proposition 1 shows that the investment ratio $\frac{N^*}{K^*}$ is higher under the expensing regime than under the capitalization regime. Each ratio can be decomposed further into a ratio of two ratios, each featuring the productivity parameter of an asset divided by the after-tax cost of providing that asset. In each case, the tangible asset has a productivity parameter to after-tax cost ratio of $\frac{\alpha}{\rho}$; the cost of using the tangible asset is simply ρK because the asset does not depreciate. In each case, the intangible asset has a productivity parameter of β. Under the capitalization regime, only the economic depreciation of the intangible asset is deductible, so the after-tax cost is $\rho + \lambda(1-\tau)$. Under the expensing regime, the after-tax cost is $(1-\tau)(\rho+\lambda)$.

A comparison of the investment ratios shows that $\frac{N^*}{K^*}$ is higher under the expensing regime than under the capitalization regime by a factor of $1+\frac{\rho\tau}{(\rho+\lambda)(1-\tau)}$. Therefore, $\frac{N^*}{K^*}$ under the capitalization regime multiplied by $1+\frac{\rho\tau}{(\rho+\lambda)(1-\tau)}$ is equal to $\frac{N^*}{K^*}$ under the expensing regime. The difference between the investment ratio under the expensing regime and the investment ratio under the capitalization regime is increasing in the tax rate τ and the project's after-tax weighted average cost of capital ρ, and is decreasing in the probability λ that the intangible assets will stop being productive.

2.3 Investment in an accounting information environment

Having established the ratio of intangible to tangible investments in the full information environment under each tax regime, we turn to a model of firm investment behavior in a setting in which accounting measures of the firm's investment decisions are of first-order importance.

The tax and financial reporting system in this section has three properties, which we collectively refer to as the *accounting information environment*. First, investments in internally developed intangible assets are expensed for tax purposes when the expenditure is made, i.e., the firm operates under the *expensing regime* for tax purposes. This is consistent with the typical tax treatment of such expenditures. Second, the accounting system exhibits *book-tax conformity* in that the investments in internally developed intangible assets are also expensed for financial reporting purposes. Third, the accounting system exhibits *income aggregation* in that the accounting system reports period one pre-tax operating income of $y = x - N$. The accounting report does not provide a decomposition of *y* into its two components, where *x* is the firm's date one operating cash flow and *N* is the firm's date zero investment in intangible assets.[3] The book tax expense on date one is τy. The accounting report issued at the moment that the first period operating cash flow is realized is a vector $\left(y(1-\tau), K\right)$, where the first element is the project's after-tax operating accounting income and the second element is the project's tangible assets.

On date zero, the firm makes its investment choices *K* and *N* at an after-tax cost of $K + N(1-\tau)$. Investors do not observe these choices. On date one, four events occur. First, the firm receives its operating cash flow *x* and pays taxes of τx, retaining the difference. Second, the firm issues the accounting report $\left(y(1-\tau), K\right)$. Third, the current investors sell their ownership interests to the next generation of investors. Current investors must sell their interests; competition among the next generation of investors forces the price to be equal to the present value of future cash flows that the project generates, conditional upon the accounting report. Fourth, the firm observes whether the project will generate cash flows in the future. If it will not, the project is abandoned and the tangible assets are sold for *K*.

To illustrate the importance of the aggregated income report of *y*, consider first the price *P* of the investors' ownership interests if the investors could observe the two

[3] In practice, there is a continuum of disclosure about a firm's investments in intangible assets. For example, some firms disclose R&D and advertising as a line item on the face of the income statement. Our model considers only two cases: full information about intangible assets or no information about intangible assets. This helps to highlight the effect of the information environment on firm's investments in intangible assets and is consistent with our expansive view of intangible assets beyond those created by R&D and advertising.

components of y, x and N. If the investors knew x, they would know that expected future operating cash flows would continue to be x for as long as the intangible assets were productive, because the expected cash flow on date $j+1$ is equal to the realized cash flow on date j. Using this information, investors would set the price of the claims to the firm's assets on date one equal to

$$P = \lambda K + \sum_{j=1}^{\infty} \frac{[(1-\tau)x + \lambda K](1-\lambda)^j}{(1+\rho)^j} = \frac{x(1-\tau)(1-\lambda) + \lambda K(1+\rho)}{\rho + \lambda}. \quad (6)$$

In this case, from the perspective of date zero, the present value of the project's future after-tax cash flows would be the sum of the expected price P from (6) and the expected date one after-tax cash flow, discounted one period. Using the relation $E(x) = \mu q(K,N)$, this amount is

$$\frac{(1-\tau)\mu q}{1+\rho} + \frac{(1-\tau)\mu q(1-\lambda) + \lambda K(1+\rho)}{(\rho+\lambda)(1+\rho)} = \frac{(1-\tau)\mu q + \lambda K}{\rho + \lambda}, \quad (7)$$

which is the same as (2). So with a disaggregated accounting report, the firm would choose the investments on date zero consistent with proposition 1(a), that presents $\frac{N^*}{K^*}$ for the full information environment under the expensing regime.

However, on date one, investors face a valuation problem because of their inability to observe x. Investors observe after-tax income $y(1-\tau)$ and tangible assets K. They know that $x = y + N$ and must infer x from the accounting report $\left(y(1-\tau), K\right)$. To do this, we assume that investors make a *conjecture* on date one that the firm invested θK in intangible assets on date zero.[4] Because y and K are reported on the firm's income

[4] Other conjectures are possible, so this equilibrium is not unique. If, for example, investors conjectured that the firm would choose $N = \eta$, the firm's investment choices would be different but the result would be qualitatively similar; the firm would underinvest in intangible assets compared to the full information environment under the expensing regime.

statement and balance sheet, respectively, investors will infer that $x = y + \theta K$. The value of θ is derived endogenously from the firm's objective of maximizing the present value of the date one operating cash flow plus the date one expected price less the after-tax cost of its investments. Of course, the conjectured value of θ must be consistent with the firm's equilibrium investment decisions.

The price on date one is

$$P = \frac{(1-\tau)(1-\lambda)E(x \mid y) + \lambda K(1+\rho)}{\rho + \lambda}. \tag{8}$$

Using the relation $E(x \mid y) = y + \theta K$, (8) simplifies to

$$P = \frac{(1-\tau)(1-\lambda)(y+\theta K) + \lambda K(1+\rho)}{\rho + \lambda}. \tag{9}$$

On date zero, the firm maximizes the present value of the date one operating cash flow plus the date one expected price less the after-tax cost of the investments. Using the relation $E[y] = \mu f(K,N) - N$ yields the following maximization problem.

$$\max_{K,N} \left\{ \frac{(1-\tau)\mu f(K,N) + \lambda K}{\rho + \lambda} + \frac{(1-\tau)(1-\lambda)(\theta K - N)}{(1+\rho)(\rho+\lambda)} - K - N(1-\tau) \right\} \tag{10}$$

The maximization problem in (10) is identical to the maximization problem in (3), except for the term $\frac{(1-\tau)(1-\lambda)(\theta K - N)}{(1+\rho)(\rho+\lambda)}$. This term captures the firm's incentive to underinvest in N and overinvest in K because the date one stock price reflects the inferred value of x, $y + \theta K$, rather than the value of x itself.

Proposition 2 presents the investment ratio $\frac{N^*}{K^*}$ associated with the solution to the firm's maximization problem characterized in (10) under the accounting information environment.

Proposition 2: In the accounting information environment,

$$\frac{N^*}{K^*} = \theta = \frac{\beta\rho(1+\rho)}{(1-\tau)[\alpha(1+\rho+\rho^2+\rho\lambda)+\beta(1-\lambda)]}.$$

The effect of the accounting information environment on the firm's investment ratio can be seen by dividing the investment ratio in proposition 2 by the investment ratio in proposition 1(a). In each case, the firm operates under the expensing regime. The difference is that the firm operates in the full information environment in proposition 1(a) but in the accounting information environment in proposition 2.

By comparing the investment ratios, $\frac{N^*}{K^*}$ from proposition 2 is smaller by a factor of $1-\frac{(\alpha+\beta)(1-\lambda)}{\alpha(1+\rho+\rho\lambda+\rho^2)+\beta(1-\lambda)}$ than $\frac{N^*}{K^*}$ from proposition 1(a). Therefore, the financial reporting treatment of investments in intangible assets induces firms to invest less in intangible assets relative to tangible assets. The key is aggregation in the financial reports. Because the income statement subtracts investments in internally developed intangible assets (*N*) from operating cash flows (*x*) to arrive at net income (*y*), the firm responds to short-term incentives to boost stock price by underinvesting in intangible assets. Note that investors are not being fooled here; the investment ratio that investors anticipate occurs in equilibrium.[5] A firm that chose, for example, the investment ratio from the full information environment in proposition 1(a) would drive down the date one price of the stock, as the unexpectedly high investment in intangibles would induce investors to underestimate the future operating cash flows *x*.

[5] The idea that managers act inefficiently, even with efficient capital markets, was first formally developed in a signal-jamming model in Stein (1989). The notion that managers make inefficient investment choices and that investors are not being fooled is also present in Kanodia et al. (2004).

The accounting information environment also provides the firm with an incentive to overinvest in tangible assets. As α approaches zero, K^* approaches zero in the full information environment but not in the accounting information environment. This occurs because investing in K increases the opportunity cost of underinvesting in N. Because investors can observe K on the firm's balance sheet, the firm is willing to overinvest in K to signal to investors that it has an incentive to invest in N in the accounting information environment.

The effect of the accounting information environment on the investment ratio is exacerbated when λ is low or when ρ low. In either case, the date one operating cash flow becomes relatively less important than the date one stock price when determining the date zero stock price, which increases the firm's incentive to underinvest in intangible assets on date zero. The effect is also exacerbated by a lower value of α or a higher value of β. The more important intangible assets are to the production process, the greater is the distorting effect of accounting aggregation on the investment ratio.

Once the effect of accounting aggregation on the investment ratio is considered, the effect of the expensing regime on firms' incentives to invest in intangible assets becomes more nuanced. As shown in proposition 1, the expensing regime induces a higher ratio of intangible to tangible assets relative to the ratio under the capitalization regime in the full information environment. However, in the accounting information environment, expensing could instead produce an investment ratio that is higher or lower than the ratio in proposition 1(b) that presents $\frac{N^*}{K^*}$ for the full information environment under the capitalization regime. This happens because the tax *advantages from expensing* and the financial reporting *disadvantages from information aggregation* have opposing effects of firm's investment in intangible assets. Proposition 3 summarizes these countervailing effects on the investment ratio.

Proposition 3: The investment ratio $\frac{N^*}{K^*}$ under the accounting information environment is higher than under the capitalization regime in the full information environment if and only if $\left[\frac{\tau}{1-\tau}\right]\left[\frac{\alpha}{\alpha+\beta}\right] > \frac{1-\lambda}{\rho(1+\rho)}$.

Whether the firm invests too much or too little in intangible assets relative to tangible assets in the accounting information environment compared to the capitalization regime and the full information environment depends on the terms $\frac{\tau}{1-\tau}$, $\frac{\alpha}{\alpha+\beta}$, and $\frac{1-\lambda}{\rho(1+\rho)}$. The first term reflects the importance of taxes. The higher the tax rate, the more likely it is that the firm will overinvest in intangible assets in the accounting information environment. The tax term is multiplied by the term $\frac{\alpha}{\alpha+\beta}$, which reflects the relative importance of tangible assets on output. Therefore, the greater the relative importance of tangible assets, the more likely it is that the firm will overinvest in intangibles. The last term reflects the importance of the date one stock price relative to the date one operating cash flow. If it is high (that is, if the first period operating cash flow is of little importance relative to date one stock price), then the firm will tend to underinvest in intangible assets in the accounting information environment, despite the tax advantage accorded intangible investments. Finally, when $\left[\frac{\tau}{1-\tau}\right]\left[\frac{\alpha}{\alpha+\beta}\right] = \frac{1-\lambda}{\rho(1+\rho)}$, the tax and financial accounting effects are exactly offsetting, and the investment ratio in the accounting information environment is equal to the investment ratio in the full information environment under the capitalization regime. In other words, the tax incentives to invest in intangible assets are completely mitigated by financial reporting disincentives to invest in intangible assets.

Overall, proposition 3 says that the claim that the tax treatment of internally developed intangible assets leads to excess investment in intangible assets need not hold in the accounting information environment. If the appropriate benchmark is the investment ratio $\frac{N^*}{K^*}$ under the capitalization regime in the full information environment, then the accounting information environment could lead to either overinvestment or underinvestment in intangible assets.

4. Tax preference measure

In this section, we propose a tax preference measure that reflects the tax advantages of both investments in intangible assets and debt financing. In each case, the tax advantage does not change the firm's *accounting effective tax rate*, defined as the ratio of book tax expense to pre-tax financial accounting income. This rate is often used as an indicator of tax-favored investments. In this setting, however, the accounting effective tax rate is of no use whatsoever. Immediate expensing of investments in internally developed intangible assets and interest payments for both financial accounting and tax purposes ensure that in every period, the accounting effective tax rate is τ in our model. This measure is unaffected by both the firm's use of funds (i.e., mix of tangible and intangible assets) and its source of funds (i.e., mix of debt and equity). The fact that the accounting effective tax rate reveals *book-tax differences* instead of *tax preferences* motivates our search for a different measure, which we call the *economic effective tax rate.*

We abstract away from the firm's choices of K and N in this section, which allows us to model the process by which assets decay in a less restrictive fashion. We also explicitly model the firm's capital structure in this section. On date zero, the firm invests K dollars in tangible assets and N dollars in intangible assets. Intangible assets decay at a constant rate λ; tangible assets decay at a constant rate δ. Both types of assets are replaced as they decay. Investments in intangible assets are expensed for tax purposes when made; investments in tangible assets are capitalized when made and depreciated at their rate of economic decay δ. Therefore, the after-tax cost of the initial investment is $K + N(1-\tau)$ and the book value of the firm's assets is K. The firm generates expected after-tax cash flows from its investments of $(1-\tau)\left[\mu f(K,N) - \delta K - \lambda N\right]$ per unit of time. This cash is distributed to debt holders in the form of interest and equity holders in the form of dividends.

We express the value of the firm using the adjusted-present-value (APV) method in which firm value is decomposed into the value of an all-equity firm and the tax savings from debt financing. The firm operates in an environment in which the value of the firm's debt (D) and equity (E) is equal to the after-tax cost of its investments.[6] Debt holders are paid the risk-free rate of interest, r, which is deductible when determining the corporate income tax. The interest payments between equity holders and debt holders do not affect firm value, but the tax savings generated from the interest deductions provide the equity

[6] This implies that the firm does not earn abnormal expected returns on its investments. A setting in which the firm earns abnormal expected returns on certain identified assets can be recharacterized as one in which the firm owns additional unidentified assets. Our definition of intangible assets is an expansive one.

holders with a risk-free stream of cash flows of τrD. The after-tax cost of risky equity capital if the firm were an all-equity firm is π. Therefore,

$$\int_0^{\infty}(1-\tau)[\mu f(K,N)-\delta K-\lambda N]e^{-\pi t}dt+\int_0^{\infty}\tau rDe^{-rt}dt \\ =K+N(1-\tau)=E+D. \tag{11}$$

An equivalent representation of the value of the firm's debt and equity would be to discount the after-tax operating cash flows at the after-tax weighted average cost of capital ρ, which would yield

$$\int_0^{\infty}(1-\tau)[\mu f(K,N)-\delta K-\lambda N]e^{-\rho t}dt \\ =K+N(1-\tau)=E+D. \tag{12}$$

We focus on the first representation to highlight the role of tax-favored debt financing on both firm value and our tax preference measure.[7]

The pre-tax rate of return R is the discount rate for which the present value of future pre-tax cash flows from the investments to all stakeholders (i.e., debt holders, equity holders, and the government) equals the pre-tax cost of the investments.

$$\int_0^{\infty}[\mu f(K,N)-\delta K-\lambda N]e^{-Rt}dt=K+N. \tag{13}$$

The right-hand side of (13) is in part financed by the government in two ways. First, the government provides a tax deduction for the initial investment in intangible assets of τN. Second, the government provides a subsidy with a present value of τD in the form of future tax savings associated with interest expense. The remainder of the investment is

[7] The APV method was introduced in Myers (1984). This valuation method calculates the net present value of the all-equity-financed project and adds the value of the tax benefits of debt. The weighted average cost of capital method accounts for any tax benefits of debt by adjusting the discount rate. The value of the tax benefits of debt in our model incorporate the assumption in Hamada (1972), which assumes that the tax shield is riskless and should be discounted back to date zero at the risk-free rate.

financed by the bondholders and stockholders. Therefore, we express the investment provided by all stakeholders as $E + D + \tau N$ and the investment provided by bondholders and stockholders as $E + D(1 - \tau)$. Using (13) and the fact that $K + N = E + D + \tau N$, we express the pre-tax rate of return R on the investment provided by all stakeholders as

$$R = \frac{\mu f(K,N) - \delta K - \lambda N}{E + D + \tau N}. \tag{14}$$

The after-tax rate of return π is the discount rate for which the present value of future after-tax cash flows from the project to private investors, $(1 - \tau)[\mu f(K,N) - \delta K - \lambda N]$, equals the after-tax cost of the investments borne by private investors, $E + D(1 - \tau)$. We use (11) to express the after-tax rate of return on investment provided by bondholders and stockholders as

$$\pi = \frac{(1 - \tau)[\mu f(K,N) - \delta K - \lambda N]}{E + D(1 - \tau)}. \tag{15}$$

The economic effective tax rate ϕ for the firm represents the total pre-tax rate of return from (14) minus the private after-tax rate of return from (15), expressed as a percentage of the total pre-tax rate of return.

$$\phi = \frac{R - \pi}{R} = \tau - \frac{\tau N(1 - \tau)}{E + D(1 - \tau)} - \frac{\tau D(1 - \tau)}{E + D(1 - \tau)}. \tag{16}$$

The upper bound for ϕ in (16) is τ. If a firm has no tax-favored intangible investments (i.e., $N = 0$) and no debt financing (i.e., $D = 0$), the economic effective tax rate and the statutory tax rate are the same. The decomposition in (16) separates the effects of intangible investments and debt financing on the firm’s economic effective tax rate. Our empirical estimate of ϕ in Section 5 shows that our tax preference measure varies by industry and over time in ways that provide important insights into the effect of these invisible tax preferences on sources and uses of firms' financing decisions.

We illustrate our model with the following example. A firm subject to a 35 percent tax rate invests \$121 million in a project on date zero. The investment in tangible capital *K* is \$81 million and the remainder is in intangible capital *N* so the after-tax cost of the initial investment is

$$\$81{,}000{,}000 + \$40{,}000{,}000(1-.35) = \$107{,}000{,}000. \tag{17}$$

The initial after-tax cost of the investment is provided by equity holders who provide \$87,000,000 and the remaining \$20,000,000 by debt holders who receive the risk-free rate of 6 percent. The cost of equity capital for an all-equity investment is 15.73 percent. The project generates an expected pre-tax cash flow in perpetuity of \$24,200,000; this amount includes amounts needed to replace tangible and intangible capital as they decay. The internal rate of return to all stakeholders is

$$R = \frac{\$24{,}200{,}000}{\$121{,}000{,}000} = 20\%. \tag{18}$$

Debt holders and equity holders together invest \$107,000,000, receiving a risky after-tax expected cash flow of \$24,200,000 (1 − .35) = \$15,730,000 and a risk-less cash flow of \$20,000,000 (.06) (.35) = \$420,000 from the tax deduction for interest expense. The present value of the riskless future tax savings is \$420,000 / .06 = \$7,000,000. Using (15),

$$\pi = \frac{\$15{,}730{,}000}{\$100{,}000{,}000} = 15.73\%. \tag{19}$$

The economic effective tax rate is

$$\phi = \frac{R-\pi}{R} = \frac{20\% - 15.73\%}{20\%} = 21.35\%. \tag{20}$$

Finally, we show that the firm's after-tax weighted average cost of capital (WACC) satisfies (12). Using (12), the weighted average cost of capital must be

$$\rho = \frac{\$15,730,000}{\$107,000,000} \approx 14.7\%.$$

By combining the Hamada (1972) model, in which both the debt and its tax shield are risk-free with the WACC equation, ρ can be expressed as a function of π, and the firm's debt to value ratio

$$\rho = \pi\left[1 - \frac{\tau D}{E+D}\right].\text{[8]} \tag{21}$$

5. Empirical estimation

In this section, we estimate our tax preference measure from (16) for the time period 1988 through 2005. *K* is the book value of the firm's assets. *D* is the book value of the firm's short-term and long-term interest-bearing debt. *E* is the market value of the firm's equity plus the book value of its non-interest bearing liabilities.[9] τ is the top statutory corporate tax rate of 34 percent for 1988-1992 and 35 percent for 1993-2005. Using the relation $K + N(1-\tau) = E + D$, we substitute $E + D - K$ for $N(1-\tau)$ in (16) for our empirical estimation. Accordingly, we proxy for intangible capital in our model as

[8] Grinblatt and Titman (2002, 482) formally prove this by showing that static perpetual debt in a WACC formula reduces to Modigliani and Miller's (1963) adjusted cost of capital formula, which gives the firm's WACC as a function of its debt to value ratio.

[9] In Compustat, K is defined as data6; D is defined as data9 + data34; E is defined as data25*data199+data6-data60-data9-data34. For financial firms, interest bearing debt (included in our measure of D) also includes data53 from the annual bank file (i.e., bna).

the difference between book value and market value of assets, consistent with our expansive definition of intangible assets.[10]

Because we use the difference between the book value and market value of firm assets as a proxy for intangible capital $N(1-\tau)$ in our model, firm-level estimates of economic effective tax rates are potentially misleading. Suppose five firms invest in an effort to be the first to discover a new drug. At the industry level, our measure recognizes the value of the discovery to the winning firm and the cost of the investments incurred by all five firms. At the firm level, however, the economic effective tax rate of the winning firm will be biased downward and the economic effective tax rates of the losing firms will be biased upward. To mitigate these effects, we estimate economic effective tax rates in (16) by using aggregate measures of *K*, *D* and *E* by industry for each year.

Our sample consists of 15,726 firms from the Compustat Industrial Annual File. We exclude 1,363 foreign firms, partnerships, and subsidiaries and 5,199 firms in untabulated industries. Table 1 reports the determination and industry makeup of the sample. We use the same industry classifications as in Barth et al. (1999).

[INSERT TABLE 1 HERE]

For the entire sample, the ratio of the book value of assets to the market value of equity plus the book value of interest-bearing debt is $\frac{K}{E+D} = 75\%.$ Using the relation

[10] We emphasize that our paper deals with investments in *internally-developed* intangible assets. When intangible assets are purchased externally, the value of these intangibles is reflected (i.e., the assets are "stepped-up") in the book value of a firm's assets if purchase accounting was used for GAAP. Where purchase accounting was used but there was no corresponding step-up in basis for tax purposes, our economic effect tax rate measure will be overstated. This is because our measure will detect fewer tax-favored investments in internally-developed intangible assets, causing the effect of intangible assets of our tax preference measure to be understated. If the accounting and tax treatments of externally purchased intangibles are the same (i.e., a step-up in basis of assets for both or neither), then our economic effective tax rate will properly reflect this tax preference.

$E + D = K + N(1 - \tau)$, this implies that the ratio of investment in intangible assets to tangible assets for our sample, $\frac{N}{K}$, is about 0.51.

Table 2, Panels A and B report our empirical estimate of the economic effective tax rate ϕ by year and by industry, respectively. We tabulate the mean ϕ for each industry and for each year in the sample and we show the decomposition of our measure as in (16). The column denoted "Effect of Intangible Assets" shows the effect of investments in intangible assets on the economic effective tax rate. The column denoted "Effect of Debt Financing" shows the effect of tax-deductible interest payments on the economic effective tax rate.

[INSERT TABLE 2 HERE]

Our results show the average value of ϕ is 16 percent, during which time the top statutory rate was either 34 percent (1988-1992) or 35 percent (1993-2005). On average, the statutory tax rate is reduced by 13 percentage points due to immediate expensing of intangible assets. The use of debt financing further reduces the statutory tax rate by 6 percentage points.

Table 2 Panel A shows a general decline in the economic effective tax rate over time. The average measure declines from 19 percent in 1988 to 15 percent in 2005. This downward trend is consistent with the growing importance of intangible assets in the economy from 1988 through 2005. Additionally, investments in intangible assets play a more significant role over time relative to debt financing in reducing the economic effective tax rate below the statutory rate.

Table 2 Panel B shows our economic effective tax rate measure by industry. The effect of investments in intangible assets on the economic effective tax rate varies across industries from 3 percent to 25 percent. This effect is highest for industries with a relatively intensive use of intangible capital – pharmaceuticals (25 percent), chemicals (16 percent), food (19 percent), and computers (18 percent). The effect is lowest for industries with a relatively intensive use of tangible and financial assets – utilities (6 percent), insurance (7 percent) and financial services (3 percent).

While the expensing of interest payments for both tax and financial reporting purposes does not lower a firm's accounting effective tax rate, debt financing is tax-favored relative to equity financing because payments to debt holders are tax deductible. While the effect of debt financing on the economic effective tax rate remained relatively stable over our time period, the effect does vary by industry.

Industries with a relatively intensive use of intangible capital have the lowest effect of debt financing on the economic effective tax rate. Table 2 Panel B shows that the smallest effect of debt financing on the economic effective tax rate occurs in the pharmaceutical and computer industries, with a 1 and 2 percentage point decrease, respectively. The largest effect of debt financing on the economic effective tax rate occurs in industries with a relatively intensive use of tangible and financial assets, such as utilities and financial services, with a 9 and 11 percentage point decrease, respectively.

The correlation coefficient of the two effects on our measure, intangible assets and debt financing, is –.903. We conjecture that this relation exists because firms with high levels of intangible assets relative to tangible assets have limited collateral against which to borrow, making debt financing potentially more costly than firms with a significant tangible asset base.[11] The use of debt financing appears to play a more significant role in reducing the economic effective tax rate for firms with a relatively intensive use of tangible or financial assets. As a result, the intra-industry variance of our measure is reduced when financing and investment are considered together, rather than just looking at the effect of intangible assets in isolation (i.e., .0008 versus .0030).

6. Conclusions

Our study of the tax and financial reporting issues associated with investment in internally developed intangible assets yields two important insights. Because investors cannot distinguish between intangible investments and operating expenses in our model, managers have an incentive to underinvest in intangibles despite their favorable tax treatment. The net effect of financial reporting costs and tax benefits could increase or decrease the ratio of investment in intangible assets to tangible assets relative to a benchmark case in which such intangible investments were capitalized for tax purposes and investors could distinguish intangible investments from operating expenses. These findings suggest that the financial reporting treatment of investments in intangible assets mitigates the tax incentives that encourage such investments.

Second, we develop a tax preference measure that detects investments in intangible assets and the use of debt financing, which are both tax-favored but do not generate book-tax differences. Our measure indicates that the economic effective tax rate was, on average, about 16 percent between 1988 and 2005. This rate varies substantially by industry, from a low of 9 percent for the pharmaceutical industry and a high of 21 percent for financial services firms. Additionally, we find that the effect of debt financing

[11] Another possible interpretation of this relation is that firms with a relatively intensive use of intangible assets for which investments are immediately expensed may find themselves in a tax loss position, which would increase the after-tax cost of interest.

on the economic effective tax rate is smaller for firms with relatively high levels of intangible assets relative to tangible assets. These findings suggest accounting-based tax preference measures fail to reflect important tax preferences. In addition, because the effects of the two tax preferences on our measure are negatively related, this suggests that any analysis of intra-industry variation in tax-favored investment should consider both the uses and sources of corporate financing.

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TABLE 1
SAMPLE DETERMINATION AND INDUSTRY DISTRIBUTION

Sample Determination	**N**
All firms on Compustat (Industrial Annual)	22,288
Partnerships, subsidiaries, foreign firms	1,363
Firms in untabulated industries	5,199
Final sample	15,726

Industry Distribution	**Standard Industrial Classification**	**N**
Mining	1000-1999, excluding 1300-1399	445
Food	2000-2111	273
Textiles	2200-2790	598
Chemicals	2800-2824, 2840-2899	269
Pharmaceuticals	2830-2836	696
Extractive	2900-2999, 1300-1399	586
Durables	3000-3999, excluding 3570-3579 and 3670-3680	2,869
Computers	7370-7379, 3570-3579, 3670-3679	2,380
Transportation	4000-4899	839
Utilities	4900-4999	358
Retail	5000-5999	1,510
Insurance	6500-6999	748
Services	7000-8999, excluding 7370-7379	1,611
Financial	6000-6411	2,544

TABLE 2
ECONOMIC EFFECTIVE TAX RATES

Panel A: By Year

$$\phi = \tau - \frac{\tau N(1-\tau)}{E + D(1-\tau)} - \frac{\tau D(1-\tau)}{E + D(1-\tau)} \qquad (16)^a$$

Year	**Statutory Tax Rate** τ	**Effect of Intangible Assets** $\frac{\tau N(1-\tau)}{E+D(1-\tau)}$	**Effect of Debt Financing** $\frac{\tau D(1-\tau)}{E+D(1-\tau)}$	**Economic Effective Tax Rate** ϕ
1988	.34	.08	.07	.19
1989	.34	.10	.07	.17
1990	.34	.08	.07	.19
1991	.34	.11	.07	.17
1992	.34	.11	.06	.17
1993	.35	.13	.06	.16
1994	.35	.12	.06	.18
1995	.35	.14	.06	.16
1996	.35	.14	.05	.16
1997	.35	.16	.05	.14
1998	.35	.16	.05	.14
1999	.35	.16	.05	.13
2000	.35	.15	.05	.14
2001	.35	.14	.06	.16
2002	.35	.12	.06	.16
2003	.35	.14	.06	.16
2004	.35	.14	.05	.15
2005	.35	.14	.06	.15
Mean	.35	.13	.06	.16

[a] We estimate $N(1-\tau)$ using the relation $E + D = K + N(1-\tau)$. *E* is the market value of a firm's equity plus the book value of its non-interest bearing liabilities. *D* is the book value of a firm's short-term and long-term debt. *K* is the book value of a firm's assets. τ is the top corporate tax rate. We compute ϕ using aggregate measures of *E, D,* and *K* by industry for each year. .

TABLE 2 (cont.)
ECONOMIC EFFECTIVE TAX RATES

Panel B: By Industry

$$\phi = \tau - \frac{\tau N(1-\tau)}{E + D(1-\tau)} - \frac{\tau D(1-\tau)}{E + D(1-\tau)} \qquad (16)^{a}$$

Industry	**Statutory Tax Rate** τ	**Effect of Intangible Assets** $\frac{\tau N(1-\tau)}{E+D(1-\tau)}$	**Effect of Debt Financing** $\frac{\tau D(1-\tau)}{E+D(1-\tau)}$	**Economic Effective Tax Rate** ϕ
Mining	.35	.12	.05	.18
Food	.35	.19	.04	.12
Textile	.35	.12	.05	.17
Chemical	.35	.16	.04	.15
Pharmaceutical	.35	.25	.01	.09
Extractive	.35	.13	.04	.18
Durables	.35	.10	.07	.18
Computers	.35	.18	.02	.14
Transportation	.35	.12	.05	.17
Utilities	.35	.06	.09	.20
Retail	.35	.15	.04	.16
Insurance	.35	.07	.12	.16
Services	.35	.13	.05	.16
Financial	.35	.03	.11	.21
Mean	.35	.13	.06	.16
Variance		0.0030	0.0009	0.0008
Correlation		-0.90299		

[a] We estimate $N(1-\tau)$ using the relation $E + D = K + N(1-\tau)$. *E* is the market value of the firm's equity plus the book value of its non-interest bearing liabilities. *D* is the book value of the firm's short-term and long-term debt. *K* is the book value of the firm's assets. τ is the top corporate tax rate. We compute ϕ using aggregate measures of *E, D,* and *K* by industry for each year. .

APPENDIX

Proof of proposition 1: (a) Differentiating (3) with respect to *K* and *N* yields the following first-order conditions where $\frac{\partial f(K,N)}{\partial K} = q_K$ and $\frac{\partial f(K,N)}{\partial N} = q_N$.

$$\frac{(1-\tau)\mu q_K}{\rho} = 1 \tag{A1}$$

$$\frac{\mu q_N}{\rho + \lambda} = 1 \tag{A2}$$

Setting the left-hand side terms from (A1) and (A2) equal to each other and substituting in the partial derivatives of (5) with respect to *K* and *N* yields the result.

(b) Differentiating (4) with respect to *K* and *N* yields the following first-order conditions.

$$\frac{(1-\tau)\mu q_K}{\rho} = 1 \tag{A3}$$

$$\frac{(1-\tau)\mu q_N}{\rho + \lambda(1-\tau)} = 1 \tag{A4}$$

Setting the left-hand side terms from (A3) and (A4) equal to each other and substituting in the partial derivatives of (5) with respect to *K* and *N* yields the result.

QED

Proof of proposition 2:

Differentiating (10) with respect to *K* and *N* yields the following first-order conditions.

$$\frac{(1-\tau)\mu q_K(1+\rho)}{\rho(1+\rho)-\theta(1-\lambda)(1-\tau)}=1 \tag{A5}$$

$$\frac{\mu q_N(1+\rho)}{1+\rho+\rho^2+\rho\lambda}=1 \tag{A6}$$

(A5) and (A6) jointly imply

$$\frac{N^*}{K^*}=\frac{\beta[\rho(1+\rho)-\theta(1-\lambda)(1-\tau)]}{\alpha(1-\tau)(1+\rho+\rho^2+\rho\lambda)}. \tag{A7}$$

The investors' conjecture that $N=\theta K$ must be confirmed in equilibrium, which implies

$$\theta=\frac{\beta\rho(1+\rho)}{(1-\tau)[\alpha(1+\rho+\rho^2+\rho\lambda)+\beta(1-\lambda)]}. \tag{A8}$$

Substituting the value of θ from (A8) into (A7) yields the result.

QED

Proof of proposition 3: Dividing the investment ratio from proposition 2 by the investment ratio from proposition 1(b) and simplifying yields the result.

QED