Revised: July 6, 2005

OPTIMAL COMMODITY TAXATION OF

TRADITIONAL AND ELECTRONIC COMMERCE

George R. Zodrow

Professor of Economics and

Rice Scholar, Baker Institute for Public Policy

Rice University

Houston, TX 77251-1892

zodrow@rice.edu

Research support from the American Tax Policy Institute is gratefully acknowledged. An earlier version of this paper was presented at a conference sponsored by ATPI in Washington, DC on July 8, 2004. I would like to thank Joseph Cordes, Charles McLure, Tom Neubig and the conference participants for many helpful comments. In addition, I would like to thank Joyce Tung and Michele Hendrix for research assistance and extend special thanks to Charles Collins of Taxware for his help with information on the costs of applying state sales and use taxes to electronic commerce.

I. INTRODUCTION

A great deal of attention has been focused in recent years on state sales taxation of electronic commerce, with numerous papers (including a December 2001 special issue of the *National Tax Journal*) and at least three high level groups (the National Tax Association Communications and Electronic Commerce Tax Project, the U.S. Advisory Commission on Electronic Commerce, and the Streamlined Sales Tax Project) exploring various aspects of this issue and a host of related problems. Views on the appropriate sales tax treatment of electronic commerce range across a broad spectrum. For example, the "majority proposal" of the Advisory Commission on Electronic Commerce (2000) would, if adopted, virtually eliminate sales taxation of remote electronic commerce (sales over the Internet from vendors outside of the taxing jurisdiction) and reduce the taxation of local electronic commerce. In particular, one recommendation would allow out-of-state online retailers to own conventional "bricks and mortar" affiliates in a state without triggering a requirement to collect use tax (a tax on sales from remote vendors that complements the sales tax), even if the affiliate accepts refunds and performs warranty work on products purchased from the online retailer. This provision could eventually result in significant revenue losses under the sales tax, especially if it were to dramatically increase the extent to which firms establish out-of-state online affiliates that take and fill orders without paying sales or use tax.

By comparison, McLure (2000) argues that the sales tax issues raised by the advent of electronic commerce merely highlight several long-standing problems of the state sales and use tax system, and that now is a propitious time to remedy these problems with a coordinated fundamental sales tax reform. He proposes a completely revamped "ideal retail sales tax" system that would be consistent with the basic principles of sales taxation long advocated by public finance economists — uniform taxation of a comprehensive consumption tax base on a destination basis, that is, with the revenues accruing to the state in which consumption occurs. Under McLure's approach, all states would agree on a common base that would include all consumption items, including services consumed by individuals. Given the common comprehensive base, each state would then be free to set its own rate. All businesses would be issued a uniform exemption certificate, and their purchases of all business inputs (defined as only those deductible under the federal income tax) would be tax exempt. Under this approach, all electronic commerce between businesses would be tax exempt, while all electronic commerce between businesses and consumers — and indeed all remote sales to consumers, including those made via mail order, telemarketing and television shopping networks — would be fully taxed. Other proposals fall between these two ends of the spectrum, including the recent model legislation proposed by the Streamlined Sales Tax Project (2001), as well as proposed treatments suggested by Fox and Murray (1997) and Mikesell (2000).

A central issue in all of these discussions is whether preferential sales tax treatment is desirable for electronic commerce. Zodrow (2000) argues that most of the arguments made in support of such preferential treatment do not survive close scrutiny — or are at least highly suspect — and are thus unlikely to justify over-riding the traditional presumption for uniform sales taxation of all consumption expenditures under a comprehensive sales tax. However, three of the arguments offered in support of preferential tax treatment of electronic commerce are potentially valid.

The first contention is that the existence of beneficial network externalities in the network of computers that form and provide access to the Internet may provide a rationale for preferential tax treatment of electronic commerce in order to encourage the expansion of the network to its efficient size. Zodrow (2002) argues that although in principle one can certainly construct a case for a tax preference for electronic commerce on network externalities grounds, it is weak under the current circumstances, especially given the recent rapid growth of the Internet. Moreover, even if such a tax preference were deemed desirable, sales tax exemptions of remote (or even all) e-commerce are a very poorly targeted, expensive and thus undesirable means of accounting for network externalities. In particular, tax exemption (or subsidization) of Internet access fees would be a much more targeted response to the issues raised by the existence of significant network externalities; note that the argument for such treatment is relatively stronger for fees for access to broadband, as the network of broadband users is still relatively small. The fact that such treatment already characterizes most US states — and because new taxes on Internet access are prohibited under the Internet Tax Freedom Act — further weakens the case for any additional preferential treatment of electronic commerce.[1] Zodrow concludes that states should adopt a goal of uniform taxation of electronic and other commerce, and focus on the difficult task of devising means of achieving that goal.[2]

A second argument is that the administrative and compliance costs associated with applying the sales tax to electronic commerce — at least from remote vendors, that is, vendors located outside the taxing jurisdiction — are especially large, so that exemption from the requirement to collect use tax is appropriate.[3] [4] Under the current sales tax system, this argument has proved to be a compelling one. In particular, the US Supreme Court has issued multiple

---

1 Of course, the desirability of exemption of Internet access fees is also open to question, especially if Internet access rates are not tax sensitive, as suggested by the evidence presented in Bruce, Deskins and Fox (2004).

2 A recent paper by Fox and Luna (2003) arrives at the same conclusion in a broader context.

3 Vendors who are deemed to have nexus within a taxing jurisdiction are required to collect tax on remote sales. However, since many Internet vendors have nexus in very few states (or sometimes in only a single state that does not have a sales tax), much of electronic commerce currently escapes sales tax. This problem is exacerbated to the extent that firms with nexus in a state can under current law set up online subsidiaries that are treated as separate entities for purposes of establishing nexus.

4 Note that these differentially high administrative costs would typically arise for tax collection by all remote vendors, including mail-order businesses and telemarketers, as well as electronic retailers, and are presumably less important for Internet sellers who have nexus in a taxing jurisdiction. Thus, although the text will follow common usage and discuss differential treatment in terms of traditional and electronic commerce, it should be remembered that the key distinction is between local and remote vendors.

rulings, most recently in its 1992 decision in *Quill Corp. v. North Dakota*, stating that requiring vendors without a physical presence (nexus) in the taxing jurisdiction to collect use tax on remote sales is sufficiently complicated that it represents an unconstitutional restriction of interstate commerce. This complexity arises primarily from the fact that sales taxes are imposed by some 8,000 different taxing entities, including state and local governments and various special taxing jurisdictions, with large variations in tax rates, bases, and administrative rules (including procedures for filing, exempting sales, registration, sourcing sales and audit). Complying with this myriad of sales and use tax regimes is difficult and costly, especially for small vendors, although software packages are currently available that greatly facilitate this effort. In addition, the anonymity that sometimes characterizes Internet transactions — customers and vendors whose locations are uncertain or perhaps even transactions conducted using anonymous "electronic cash" accounts — complicates enforcement of sales taxation even if it is legally required. The problem faced by tax authorities in monitoring such transactions is exacerbated if the transactions involve goods that can be transmitted electronically.

Note, however, that the practical importance of these problems is not obvious. For example, most transactions involve credit card purchases from well-established vendors, and widespread use of traceable electronic transactions may in fact facilitate tax collection (Bird, 2003). In addition, firms can purchase — or state governments can provide (free or at nominal cost) — computer software that will calculate tax liability for remote sales, and firms with little in the way of remote sales would be exempt under the de minimis rules provided by most proposals to tax remote sales. Moreover, the US states are currently involved in an ambitious effort — the Streamlined Sales Tax Project (SSTP) — designed to simplify and unify the state sales tax system while allowing states flexibility in setting rates, presumably in the hope that a sufficiently uniform system will prompt Congress or the Supreme Court to require the collection of use tax by remote vendors.[5] Nevertheless, it is certainly possible that imposing sales taxation on electronic commerce is — and will continue to be for the foreseeable future — somewhat more costly than taxing sales from "bricks and mortar" establishments located within a taxing jurisdiction, and that this potentially provides a rationale for preferential tax treatment.

Finally, preferential treatment of electronic commerce may be desirable due to optimal commodity tax considerations. As is well known from this literature, a uniform commodity tax structure is generally not optimal, so that such preferential treatment is certainly a possibility from a theoretical standpoint (Auerbach and Hines, 2002). However, a consensus has not yet emerged in the literature on whether the standard optimal taxation arguments for differential taxation imply that preferential treatment of electronic commerce is desirable. For example, in a comprehensive review of the optimal taxation literature and its application to the taxation of electronic commerce, Bruce, Fox and Murray (2003) stress that differential commodity taxes arise primarily due to differences in relative substitutability with untaxed leisure, and that this substitutability will assuredly be quite similar for commodities that differ only in whether they are purchased online or in a traditional retail outlet. Note that this efficiency argument for differential taxation arises not only for leisure but for any goods that are untaxed, including those that are exempt for distributional or social reasons. Bruce, Fox and Murray also note that purchases over the Internet may be more elastic than similar purchases at a traditional retail outlet so that preferential treatment may in theory be desirable, since optimal tax rules tend to prescribe low tax rates for goods with high own-price demand elasticities. Finally, they observe that equity considerations are likely to imply relatively high taxes on electronic commerce since

---

5 See McLure (2002) for a description and evaluation of the SSTP.

the individuals who purchase over the Internet tend to be relatively high income,[6] and that administrative and compliance cost issues should also be factored in optimal tax calculations.

With respect to the issue of the effects of differences in own-price elasticities for goods sold over the Internet, Goolsbee and Zittrain (1999, p. 418), drawing especially on the results of Sandmo (1981) who analyzes optimal labor income taxation in the presence of tax evasion, suggest that "if the price elasticities of Internet customers and retail customers are very different it may actually be efficient to allow those with high elasticities to have lower rates … the least distortive tax would be the one with high rates on those people who would not change their behavior." That is, if individuals who are Internet customers have relatively high own price elasticities for goods purchased online, efficiency considerations suggest that they should be taxed at relatively lower rates.[7] Goolsbee and Zittrain go on to note that since the empirical evidence suggests that Internet customers are in fact highly sensitive to tax factors (Goolsbee 2000a), preferential treatment for electronic commerce might well be appropriate on optimal commodity taxation grounds.

It must be stressed, however, that all of these arguments are fairly conjectural since, as observed by Bruce, Fox and Murray (2003, p. 27), "Despite the large number of articles in the optimal tax literature that have addressed the issue of neutral commodity taxation, none have addressed the idea of non-neutral taxation of two units of the same good which are obtained for final consumption via different modes of purchase." The purpose of this paper is to attempt to

---

6 Note that this line of reasoning is diametrically opposed to the "digital divide" argument which contends that preferential tax treatment of electronic commerce is desirable because it should facilitate use of the Internet by lower-income individuals. McLure and Hellerstein (2004) argue convincingly that this argument is not compelling, as preferential tax treatment for electronic commerce is an extremely poorly targeted and thus very expensive means of promoting use of the Internet by low-income individuals or achieving any distributional goals. Accordingly, the digital divide argument is not considered in the model utilized in this paper.

7 This argument could be rejected on equity grounds – that consumer sensitivity to price differences attributable to avoiding tax should not be considered in an optimal commodity tax calculation. For example, although it is well known that income from self employment is considerably more susceptible to tax evasion than other forms of

fill this void in the literature, and shed some additional light on the question of whether the theory of optimal commodity taxation, taking into account equity concerns, the relatively high own-price elasticities of individuals who are Internet shoppers, and potentially differentially high costs of administering taxation of electronic commerce, implies that preferential treatment of electronic commerce is desirable.

The analysis adapts the standard optimal commodity taxation model (as described in detail in Auerbach and Hines (2002) in their review of the optimal taxation literature), to consider multiple goods that are close substitutes in their physical attributes, but are less than perfect substitutes because they differ in terms of their mode of purchase and other attributes such as ease of shopping, purchase and return, ability to "touch and feel" the good, and delivery costs, times and uncertainties, etc. In addition, the standard model is modified (1) to include distributional concerns, following Diamond (1975), (2) to consider an economy in which only a subgroup of individuals purchases goods over the Internet, with the demand for such goods exhibiting a relatively high own-price elasticity, following the argument of Goolsbee and Zittrain (1999) and the model of Sandmo (1981) where only a subgroup of the population works in the informal labor market, and (3) to include differentially high costs of administering and complying with sales taxation of goods purchased over the Internet, following Kaplow (1990) in extending the analysis of Sandmo (1981) to the case of sales taxation.

It should be noted that the standard optimal taxation analysis adopted in this paper assumes that the commodity tax rates applied to both traditional and electronic commerce are perfectly flexible. In practice, the critical issue is much more likely to be whether electronic commerce (involving remote vendors) should be entirely tax exempt or should be taxed at the

---

income, preferential income tax treatment for the self-employed on such grounds would be viewed as highly inequitable (Zodrow, 2000). Nevertheless, for the sake of argument, this point is ignored in the analysis.

same rate as traditional commerce, rather than whether electronic commerce should receive a preferential rate.[8] Nevertheless, optimal taxation analysis can be quite informative to the policy debate. In particular, if the optimal tax rates for electronic commerce tend to cluster either around the standard rate applied to traditional commerce or around zero, then the analysis would provide strong support for full taxation or tax exemption, respectively.

The paper is organized as follows. The following section provides a brief overview of the literature, particularly the optimal taxation literature, as it applies to the question of the appropriate commodity taxation of traditional and electronic commerce; this discussion draws heavily on the excellent recent survey by Bruce, Fox and Murray (2003). Section III describes the theoretical optimal commodity taxation model, outlined above, that is applied to analyze the question of the desirability of preferential sales tax treatment of electronic commerce. Section IV provides some illustrative calculations of the key result of the optimal commodity taxation model analysis, attempting to shed some light on the relative magnitudes of the various effects, as well as the central issue of whether, on balance, preferential taxation or tax exemption of electronic commerce seems likely to be desirable. A concluding section summarizes the results.

II. AN OVERVIEW

The optimal taxation of goods and services has long been a central issue of the optimal taxation literature (Auerbach and Hines, 2002; Bruce, Fox and Murray, 2003). Two basic results are especially relevant to the structure of sales taxation. The first is the "production neutrality theorem" which argues that, under the appropriate conditions, taxes should not be assessed on production inputs. The basic intuition behind this result is that an appropriately designed set of

---

8 In particular, legal issues involving the Commerce Clause as well as provisions requiring uniform taxation in many state constitutions are likely to preclude preferential rates in many cases.

taxes exclusively on consumption goods can achieve any outcome that would obtain under taxation of production inputs, but taxes on consumption goods avoid the distortions of input choices that arise with taxes on production goods. Although, as will be discussed below, this result is subject to several important qualifications, it suggests the presumption that business inputs, including services consumed by businesses, should be exempt from state sales taxes, which should be applied only to consumption expenditures. Relative to this criterion, existing sales taxes in the US are far from optimal. Although all states make some attempt to exempt the purchases of business inputs from tax – for example, through exemptions of goods purchased for resale or goods that become components of other goods – these efforts are haphazard, and in practice a wide range of business inputs is subject to tax. Indeed, Ring (1999) estimates that on average only 60 percent of the base of state sales taxes consists of consumer purchases.

Of course, as stressed by Bruce, Fox and Murray (2003), the production neutrality result rests on several strong assumptions, and sales taxation of production inputs may be desirable to the extent that these assumptions are not satisfied.[9] In particular, production neutrality requires perfect competition and constant returns to scale (or the existence of a separate tax on pure profits) as well as a full set of optimal commodity taxes. Thus, taxation of business inputs may be desirable if designed to offset imperfections in input or product markets (especially if opportunities for production substitution are limited or inputs are supplied inelastically (Konishi, 1990)) or to tax indirectly consumption goods that are not subject to sales tax. The importance of the first qualification depends to the prevalence of imperfect markets. Although this is a contentious issue, many firms within a state arguably have limited market power in most national and international markets. In any event, it seems unlikely that the sales taxation of business in

[9] See also Slemrod (1990) for a discussion of this result, which is based on the work of Diamond and Mirrlees (1971).

most states is, or could reasonably be expected to be, designed to differentially tax firms with market power, and such a goal would in any case seem to be better served with specific excise taxes. The taxation of business inputs could also act to offset the sales tax exemption (or taxation at suboptimal rates) of certain final consumption commodities, especially the services provided from owner-occupied housing and a host of lightly taxed or exempt consumer services.[10] In any case, sales taxation of production inputs is an issue that will not be addressed in the analysis in this paper,[11] which will focus tax solely on the optimal taxation of consumption commodities.[12]

The second fundamental result of the optimal commodity taxation literature, which is the focus of this paper, concerns the optimal taxation of consumption commodities, and thus addresses the issue of whether uniform or differential sales taxation is desirable. It is possible to construct scenarios under which uniform taxation of all consumption commodities is optimal, even purely from an efficiency perspective. The key factor is the interaction between the demands for the various taxable commodities and the demand for untaxed leisure. If consumer tastes are such that leisure and consumption commodities are separable, then uniform taxation is desirable to avoid distortions in consumer choices across the various consumption commodities – the standard case for uniform treatment. This tendency toward uniformity is reinforced by several practical considerations. Differential tax rates are difficult to administer, especially since they inevitably raise extremely troublesome classification issues and create incentives for tax avoidance and evasion. Moreover, "real world" commodity tax differentials are more likely to

---

10 Bruce, Fox and Murray (2003, pp. 31-32) demonstrate that the taxation of intermediate inputs is significantly higher for services than for retail trade or manufacturing industries.

11 The analysis thus also ignores any distortionary effects of sales taxation of business location decisions, an issue that is addressed in Zodrow (1999).

12 Note that an alternative approach to the one utilized in this paper, left to future research, would be to treat goods purchased over the Internet and from traditional retailers as identical consumption goods, and to extend the production efficiency theorem to the case in which these goods are characterized by differences in delivery and/or acquisition costs.

reflect political factors than optimal tax considerations, and thus reduce rather than enhance economic efficiency. As emphasized by Slemrod (1990), adherence to the principle of uniform taxation thus provides a critical "anchor" against the entreaties of special interests for preferential (and typically highly inefficient) tax treatment.[13]

However, if leisure and consumption commodities are not separable goods, then a central message of the optimal taxation literature is that efficiency considerations imply some commodity tax differentials are desirable in order to indirectly tax leisure – that is goods that are relatively complementary with leisure should be taxed at higher rates than those that are more substitutable with leisure – although the extent of tax differentiation is still limited by the efficiency costs of distortions of choices among the other consumption commodities (Corlett and Hague, 1953).[14] This result, however, suggests that preferential taxation of electronic commerce will be undesirable – to the extent that goods purchased over the Internet are close substitutes for goods purchased from traditional retailers, their cross price elasticities with leisure should be similar, which implies that their tax treatments should be similar as well.[15]

Instead, the optimal taxation argument for preferential treatment of electronic commerce suggested by Goolsbee and Zittrain (1999) is based on empirical evidence which suggests that consumer demand for purchases over the Internet is highly elastic (Goolsbee, 2000a).[16] They

---

13 For further discussions of the practical difficulties of implementing commodity tax differentials and the administrative case for uniform taxation, see Harberger (1990) and Slemrod (1990).

14 Another implication of this same analysis is that the commodity tax system should be designed to cause equiproportionate reductions in compensated demands for all taxable goods, which tends to imply that goods with relatively highly own price elasticities of demand should face relatively low tax rates. This point is discussed further below.

15 Note again that this argument applies to any untaxed goods, including those that are exempt for distributional or social reasons.

16 This is not surprising, as access to the Internet effectively provides access to "cross-border" shopping to all users, and many empirical studies have shown that individuals who live along borders are highly sensitive to cross-border tax differentials (Goolsbee, 2000a). Bruce, Fox and Murray (2003) note that an additional explanation for why goods purchased over the Internet may be relatively price elastic is that delivery uncertainties are greater for such goods, so that consumers are more likely to order price elastic goods over the Internet rather than price inelastic necessities. Note, however, that some recent work has found much lower own-price demand elasticities for Internet goods than those estimated by Goolsbee (Alm and Melnick, 2003).

then suggest, by analogy to the work on optimal labor income taxation in the presence of tax evasion by Sandmo (1981), that preferential tax treatment of highly elastic purchases over the Internet may be desirable. The Sandmo model considers optimal linear income taxation in a model with two groups of individuals, one of which may engage in work in an informal labor market where the tax on labor income can be collected only at a differentially high administrative cost. Sandmo shows that, apart from equity concerns, a relatively low (expected) tax rate on labor income in the tax-evading informal sector may be desirable on efficiency grounds if the own-price supply elasticity of labor to that sector is sufficiently large.[17]

By analogy, Goolsbee and Zittrain (1999) argue that relatively low tax rates on electronic commerce may be desirable, again apart from equity concerns, since those individuals who avoid tax by purchasing goods over the Internet tend to be highly price responsive. The argument is that high taxes on electronic commerce will tend to significantly distort the behavior of individuals who are Internet shoppers and thus will have a high efficiency cost, relative to taxation of sales from traditional retailers, where the elasticity of demand is presumably lower.

The results discussed thus far focus on efficiency issues. However, equity concerns, based on the notion that the social marginal utility of income declines with income, have also been incorporated into the optimal commodity taxation literature (Diamond, 1975). The central message of this literature is that, in addition to the efficiency considerations discussed above, commodity tax rates should be differentially high on goods consumed disproportionately by those with relatively high incomes. Goolsbee and Zittrain (1999) demonstrate that individuals who shop on the Internet tend to be high income and relatively well educated; their data indicate that Internet shoppers on average have approximately $22,000 more in annual income and nearly

[17] Specifically, the absolute value of the own price elasticity of supply of labor to the informal sector must exceed the cross-price elasticity of labor supply to the regular market with respect to the net wage in the informal market.

two more years of education. However, they also note that this differential is declining over time. Thus, equity concerns are likely to imply that differentially high tax rates on electronic commerce will be appropriate, with this distinction declining over time.

Finally, as noted above, there is reason to believe that the administrative costs associated with collecting taxes from remote vendors, including those engaged in electronic commerce, may be larger than those associated with collecting taxes on sales from bricks and mortar merchants. Unfortunately, there are no data on (1) the magnitude of these administrative and collection cost differentials, (2) how such costs might decline if the state sales tax system were to become simplified along the lines envisioned by the Streamlined Sales Tax Project (or the more comprehensive reforms suggested by McLure (2000)), or (3) how these costs vary with the extent to which electronic commerce transactions are taxed.

Despite these uncertainties, differential costs of administration can be incorporated into an optimal commodity taxation analysis, following the approaches of Heller and Shell (1971), Yitzhaki (1979) and Kaplow (1990). These models suggest that optimal tax prescriptions, including the pattern of commodity taxation and especially the desirability of a broad tax base in the presence of differential administrative costs and the undesirability of taxing production inputs, will be affected once administrative costs are considered.[18]

This brief and necessarily selective review of the literature suggests the nature of the model utilized in this analysis to examine the desirability of preferential treatment of electronic commerce. Specifically, the standard optimal commodity taxation model will be modified to include (1) goods that can be purchased only over the Internet, which are close but not perfect substitutes for the same goods purchased from traditional retailers, (2) two types of consumers, one of which purchases goods over the Internet and has a relatively high own-price demand elasticity for such purchases, (3) exempt consumption goods, (4) distributional concerns, with Internet consumers characterized by relatively high incomes, and (5) potentially higher administrative costs of collecting tax on goods sold over the Internet. Note that in the construction of the model, the assumptions are generally chosen so as to accommodate the various claims of proponents of preferential treatment of electronic commerce; that is, these assumptions are chosen not because they are necessarily the most appropriate way to construct the model, but to maximize the likelihood that preferential treatment of electronic commerce will be desirable. Nevertheless, to anticipate the conclusions of the analysis, even in this context the optimal taxation of traditional and electronic commerce is much more likely to involve uniform tax treatment than tax exemption of the latter. The details of this model are provided in the following section.

III. THE MODEL

The model is a modified version of the standard single-period optimal commodity taxation model with fixed producer prices described in Auerbach and Hines (2002).[19] Individuals are assumed to maximize a standard utility function defined over several consumption goods and leisure. There is no source of exogenous income and lump sum taxes are not allowed. There are two types of individuals in the economy, A and B, and the number of each type of individual, N^A and N^B, is fixed. Individuals of type A never purchase goods over

---

18 Again, however, it should be noted that commodity tax differentials based on differential administrative costs could be viewed as highly inequitable; however, the analysis below will, for the sake of argument, neglect this concern and incorporate differential administrative costs into the analysis.

19 See the appendix for details of the derivations. The model of consumer behavior is also generally similar to that used by Alm and Melnick (2003) in their empirical analysis of the sensitivity of the demand for Internet goods to preferential tax treatment, except that it treats traditional goods and Internet goods as close rather than perfect substitutes and allows for leisure and a separate good that is exempt from sales tax for distributional or social reasons.

the Internet, while individuals of type B are frequent users of the Internet and often make purchases on line. These rather stark assumptions both simplify the model and allow for clear analogies to the similar model constructed by Sandmo (1981). However, they admittedly are fairly stringent, especially since one would expect that the number of individuals making online purchases (as well as the set of goods sold over the Internet as defined below) would increase over time.

In addition to leisure, which is the numeraire with a fixed price w, there are four types of consumer goods in the economy. Goods x_E are exempt from commodity taxation, either for distributional reasons (they are consumed disproportionately by the poor), social reasons (e.g., health care, education), administrative reasons (e.g., some services), political reasons (e.g., services from owner-occupied housing), or some combination of the above. Goods x_N are never sold on the Internet, either for technical or historical reasons, and are not closely substitutable for goods sold over the Internet. The remaining two categories of goods were the focus of the discussion above — goods that are sold both by traditional or "bricks and mortar" retailers and over the Internet. The model assumes, following the argument made by proponents of preferential treatment of electronic commerce, that these goods can be treated as separate commodities. Specifically, goods x_I are assumed to be sold only on the Internet, while goods x_S are sold only by traditional retailers. These two goods are assumed to be close but not perfect substitutes, differing only in mode of purchase and other attributes such as the nature and ease of shopping, purchase and return, the ability to "touch and feel" the good, and delivery costs, times and uncertainties, etc. Because of their similarities, these two goods (x_S and x_I) are assumed to have very similar cross-price elasticities with the other two types of goods and with leisure.

Consumer prices for these four consumption goods are denoted as

$$p_i, \ i = E, N, S, I \ .$$

Producer prices are assumed to be fixed, and denoted as

$$q_i, \;\; i = E, N, S, I \; .$$

There are two commodity tax (i.e., retail sales tax) rates in the model — t is applied to the taxable goods sold by traditional retailers (x_N and x_S) and τ is applied to the goods sold over the Internet (x_I). Goods x_E and leisure are untaxed. Under a uniform tax on (taxable) commodities, τ=t, while preferential tax treatment of electronic commerce implies that τ<t.

The N^A individuals of type A thus have an indirect utility function

$$V^A(w, q_E, q_N + t, q_S + t)$$

while the N^B individuals of type B have an indirect utility function

$$V^B(w, q_E, q_N + t, q_S + t, q_I + \tau) \; .$$

The government must raise a fixed amount of revenue, R, using only the two sales tax instruments t and τ.[20]

Raising revenues from sales taxation also results in administrative and compliance costs. The nature of these costs, in particular the differences in the functional relationships between these costs and the tax revenues raised from traditional and electronic vendors, is unclear, although one certainly would expect that tax compliance costs would exhibit declining marginal costs for both traditional and electronic vendors. The common perception, discussed above, is that such costs would be significantly higher for e-commerce due to the need to comply with use taxes and regulations in many different jurisdictions and because a disproportionately large fraction of e-commerce sales are attributable to relatively small vendors, and this may very well be the case. On the other hand, with the appropriate sales and use tax compliance software, these

[20] In general, the addition of an income tax would make it less likely that differential commodity taxation would be desirable (Saez, 2000, 2002; Kaplow, 2004). However, as noted above, distributional concerns tend to suggest that

differences may be small especially for larger vendors (and many smaller vendors should be excluded by de minimis rules). Indeed, for firms – even relatively small ones – engaged purely in electronic commerce who already have accurate records on mailing and billing addresses on all their customers for all sales, the additional costs of collecting use taxes may be less than the costs of sales tax compliance for a traditional retailer.[21] In any case, the analysis in this paper will assume that the marginal costs of compliance are either the same or greater for electronic vendors than for traditional vendors. Specifically, total administrative costs (C_t) of taxing goods sold by traditional retailers are assumed to be a function of the total revenue raised, with marginal costs (c_t) positive but declining with revenues, or

$$C_t = C_t(R_t) = C_t[t(N^A x_N^A + N^B x_N^B + N^A x_S^A + N^B x_S^B)]$$
$$c_t = \frac{dC_t}{dR_t} > 0,\ c_{tt} < 0 \ .$$

Similarly, suppose that administrative costs for taxing Internet sales (C_τ) are a (different) function of total revenue raised, also with positive but declining marginal costs, or

$$C_\tau = C_\tau(R_\tau) = C_\tau(\tau N^B x_I^B)$$
$$c_\tau = \frac{dC_\tau}{dR_\tau} > 0,\ c_{\tau\tau} < 0 \qquad ,$$

preferential taxation of electronic commerce is not desirable. Thus, if distributional concerns were fully addressed with a personal income tax, one argument against such a tax preference would be eliminated.

21 A separate issue is the one-time set-up costs that would be incurred by currently tax exempt remote electronic vendors, which are ignored in the analysis in this paper (which, as noted above, does not explicitly consider the issue of whether electronic commerce should be taxed at all but rather the question of whether it should receive a significantly preferential rate). These fixed set-up costs should presumably be amortized over many future years (and arguably compared to similarly amortized fixed costs already incurred by traditional retailers in complying with existing sales taxes), but in any case typically would not appear in the optimal tax conditions derived below, which are functions of marginal costs. In any case, the omission of these fixed costs is not particularly problematic, since they typically would range from one to three years of annual software costs, so that their amortized value would not be large relative to annual costs. Moreover, set-up costs are largest for relatively small firms without existing enterprise software, many of which would be exempt from the requirement to collect use tax due to de minimis rules. For larger firms, especially those with enterprise software from an established vendor (e.g., Oracle or SAP), set-up costs would be at the lower end of this range. (These rough approximations were generously provided by Mr. Charles Collins of Taxware.) Note also that these figures suggest that tax compliance costs for firms engaging in electronic commerce – like tax compliance costs in general – result in a modest barrier to entry for the industry.

and that marginal costs may be higher than for goods purchased from traditional retailers ($c_\tau \geq c_t$).

Following the standard optimal commodity taxation approach with heterogeneous consumers (Diamond, 1975), the government is assumed to maximize a weighted sum of individual utilities, where distributional weights γ^A and γ^B reflect the marginal social valuation of utility changes for the two individuals. Thus, the Lagrangian for the government is

$$L = N^A\gamma^A V^A(w, q_E, q_N + t, q_S + t) + N^B\gamma^B V^B(w, q_E, q_N + t, q_S + t, q_I + \tau) + \lambda \left\{ \begin{array}{l} t(N^A x_N^A + N^B x_N^B + N^A x_S^A + N^B x_S^B) + \tau N^B x_I^B \\ -\{R + C_t[t(N^A x_N^A + N^B x_N^B + N^A x_S^A + N^B x_S^B)] + C_\tau \tau [N^B x_I^B] \end{array} \right\} \quad (1)$$

Recalling the assumption of fixed producer prices, the first order conditions for the two government choice variables, the tax rates t and τ, are

$$\begin{aligned} \frac{\partial L}{\partial t} &= N^A\gamma^A\left(\frac{\partial V^A}{\partial p_N} + \frac{\partial V^A}{\partial p_S}\right) + N^B\gamma^B\left(\frac{\partial V^B}{\partial p_N} + \frac{\partial V^B}{\partial p_S}\right) \\ &\quad + \lambda(1-c_t)\left\{ \begin{array}{l} (N^A x_N^A + N^B x_N^B + N^A x_S^A + N^B x_S^B) \\ + tN^A\left(\frac{\partial x_N^A}{\partial p_N} + \frac{\partial x_N^A}{\partial p_S} + \frac{\partial x_S^A}{\partial p_N} + \frac{\partial x_S^A}{\partial p_S}\right) + tN^B\left(\frac{\partial x_N^B}{\partial p_N} + \frac{\partial x_N^B}{\partial p_S} + \frac{\partial x_S^B}{\partial p_N} + \frac{\partial x_S^B}{\partial p_S}\right) \end{array} \right\}, \\ &\quad + \lambda(1-c_\tau)\tau N^B\left(\frac{\partial x_I^B}{\partial p_N} + \frac{\partial x_I^B}{\partial p_S}\right) = 0 \\ \frac{\partial L}{\partial \tau} &= N^B\gamma^B\frac{\partial V^B}{\partial p_I} + \lambda(1-c_t)tN^B\left(\frac{\partial x_N^B}{\partial p_I} + \frac{\partial x_S^B}{\partial p_I}\right) + \lambda(1-c_\tau)\left[N^B x_I^B + \tau N^B\left(\frac{\partial x_I^B}{\partial p_I}\right)\right] = 0 \end{aligned}$$

Substituting from Roy's identity

$$-\frac{\partial V^h}{\partial p_k} = \alpha^j x_k^j, h = A, B \text{and} \quad k = N, S, I, \quad \text{where } \alpha^h = \frac{\partial V^h}{\partial M^h} \quad ,$$

into the first order conditions and collecting terms yields

$$-\frac{\lambda(1-c_t)-\gamma^A\alpha^A}{\lambda}N^A(x_N^A+x_S^A)-\frac{\lambda(1-c_t)-\gamma^B\alpha^B}{\lambda}N^B(x_N^B+x_S^B)$$

$$= (1-c_t)t\left[N^A\left(\frac{\partial x_N^A}{\partial p_N}+\frac{\partial x_N^A}{\partial p_S}+\frac{\partial x_S^A}{\partial p_N}+\frac{\partial x_S^A}{\partial p_S}\right)+N^B\left(\frac{\partial x_N^B}{\partial p_N}+\frac{\partial x_N^B}{\partial p_S}+\frac{\partial x_S^B}{\partial p_N}+\frac{\partial x_S^B}{\partial p_S}\right)\right]$$

$$+ (1-c_\tau)\tau N^B\left(\frac{\partial x_I^B}{\partial p_N}+\frac{\partial x_I^B}{\partial p_S}\right)$$

$$-\frac{\lambda(1-c_\tau)-\gamma^B\alpha^B}{\lambda}N^B x_I^B=(1-c_t)tN^B\left(\frac{\partial x_N^B}{\partial p_I}+\frac{\partial x_S^B}{\partial p_I}\right)+(1-c_\tau)\tau N^B\left(\frac{\partial x_I^B}{\partial p_I}\right)$$

Substituting from the Slutsky equation

$$\frac{\partial x_i^h}{\partial p_j}=S_{ij}^h-x_j^h\frac{\partial x_i^h}{\partial M^h},\quad S_{ij}^h<0 \text{ and } \frac{\partial x_i^h}{\partial M^h}>0 \text{ if } x_i^h \text{ is a normal good,}$$

and from the definition of total consumption of the three taxed goods

$$x_N=N^A x_N^A+N^B x_N^B;\quad x_S=N^A x_S^A+N^B x_S^B;\quad x_I=N^B x_I^B$$

into the right hand sides of the two first order conditions and collecting terms yields

$$(1-c_t)t\ [N^A\ (S_{NN}^A+S_{NS}^A+S_{SN}^A+S_{SS}^A)+N^B\ (S_{NN}^B+S_{NS}^B+S_{SN}^B+S_{SS}^B)]+(1-c_\tau)\tau N^B(S_{IN}^B+S_{IS}^B)$$

$$= N^A\ x_N^A\left[\frac{\gamma^A\alpha^A}{\lambda}+(1-c_t)t\left(\frac{\partial x_N^A}{\partial M^A}+\frac{\partial x_S^A}{\partial M^A}\right)\right]+N^A x_S^A\left[\frac{\gamma^A\alpha^A}{\lambda}+(1-c_t)t\left(\frac{\partial x_N^A}{\partial M^A}+\frac{\partial x_S^A}{\partial M^A}\right)\right]$$

$$+N^B\ x_N^B\left[\frac{\gamma^B\alpha^B}{\lambda}+(1-c_t)t\left(\frac{\partial x_N^B}{\partial M^B}+\frac{\partial x_S^B}{\partial M^B}\right)+(1-c_\tau)\tau\frac{\partial x_I^B}{\partial M^B}\right]$$

$$+N^B\ x_S^B\left[\frac{\gamma^B\alpha^B}{\lambda}+(1-c_t)t\left(\frac{\partial x_N^B}{\partial M^B}+\frac{\partial x_S^B}{\partial M^B}\right)+(1-c_\tau)\tau\frac{\partial x_I^B}{\partial M^B}\right]$$

$$-(1-c_t)(x_N+x_S)$$

$$(1-c_t)tN^B\,(S_{NI}^B+S_{SI}^B)+(1-c_\tau)\tau N^B S_{II}^B$$

$$= N^B x_I^B\left[\frac{\gamma^B\alpha^B}{\lambda}+(1-c_t)t\left(\frac{\partial x_N^B}{\partial M^B}+\frac{\partial x_S^B}{\partial M^B}\right)+(1-c_\tau)\tau\frac{\partial x_I^B}{\partial M^B}\right]-(1-c_\tau)x_I\quad .$$

Define the net social marginal valuations of income to individuals A and B, including the additional net tax revenues, taking into account administrative costs, generated from purchases of taxed goods financed with the additional income, as

$$\begin{aligned}\beta^A &= \gamma^A\alpha^A + \lambda(1-c_t)t\left(\frac{\partial x_N^A}{\partial M^A}+\frac{\partial x_S^A}{\partial M^A}\right)\\ \beta^B &= \gamma^B\alpha^B + \lambda(1-c_t)t\left(\frac{\partial x_N^B}{\partial M^B}+\frac{\partial x_S^B}{\partial M^B}\right)+\lambda(1-c_\tau)\tau\frac{\partial x_I^B}{\partial M^B}\end{aligned} \tag{2}$$

Substituting and collecting terms, the two first order conditions can be written

$$\begin{aligned}&\{(1-c_t)t[N^A\left(S_{NN}^A+S_{SN}^A\right)+N^B(S_{NN}^B+S_{SN}^B)]+(1-c_\tau)\tau N^B S_{IN}^B\}\\ &\quad+\{(1-c_t)t[N^A\left(S_{NS}^A+S_{SS}^A\right)+N^B(S_{NS}^B+S_{SS}^B)]+(1-c_\tau)\tau N^B S_{IS}^B\}\\ &=-\left[(1-c_t)(x_N+x_S)-\frac{\beta^A}{\lambda}\frac{N^A x_N^A}{x_N}x_N-\frac{\beta^B}{\lambda}\frac{N^B x_N^B}{x_N}x_N-\frac{\beta^A}{\lambda}\frac{N^A x_S^A}{x_S}x_S-\frac{\beta^B}{\lambda}\frac{N^B x_S^B}{x_S}x_S\right]\end{aligned}$$

$$\begin{aligned}&(1-c_t)tN^B(S_{NI}^B+S_{SI}^B)+(1-c_\tau)\tau N^B S_{II}^B\\ &=-\left[(1-c_\tau)x_I-\frac{\beta^B}{\lambda}\frac{N^B x_I^B}{x_I}x_I\right]\end{aligned}$$

Define β_N, β_S and β_I as the social marginal utility of revenues obtained from taxing the three taxable goods, that is

$$\begin{aligned}\beta_N &= \frac{N^A x_N^A}{x_N}\beta^A+\frac{N^B x_N^B}{x_N}\beta^B\\ \beta_S &= \frac{N^A x_S^A}{x_S}\beta^A+\frac{N^B x_S^B}{x_S}\beta^B\\ \beta_I &= \frac{N^B x_I^B}{x_I}\beta^B=\beta^B\ .\end{aligned} \tag{3}$$

Then, the two first order conditions can be written

$$(1-c_t)t[N^A(S^A_{NN}+S^A_{SN})+N^B(S^B_{NN}+S^B_{SN})]+(1-c_\tau)\tau N^B S^B_{IN}$$
$$+(1-c_t)t[N^A(S^A_{NS}+S^A_{SS})+N^B(S^B_{NS}+S^B_{SS})]+(1-c_\tau)\tau N^B S^B_{IS}$$
$$=-\frac{\lambda(1-c_t)-\beta_N}{\lambda}x_N-\frac{\lambda(1-c_t)-\beta_S}{\lambda}x_S$$

$$(1-c_t)t\,N^B(S^B_{NI}+S^B_{SI})+(1-c_\tau)\tau N^B S^B_{II}=-\frac{\lambda(1-c_\tau)-\beta_I}{\lambda}x_I$$

Letting

$$t_j=t,\ j=N,S\ and\ t_j=\tau,\ j=I$$

the compensated price elasticities are defined as

$$\varepsilon^h_{ij}=S^h_{ij}p_j/x^h_i=S^h_{ij}(q_j+t_j)/x^h_i \quad \text{or} \quad S^h_{ij}=\varepsilon^h_{ij}x^h_i/(q_j+t_j)$$

And, if units in the initial equilibrium are chosen so that all the producer prices q are equal to one, then

$$S^h_{ij}=\varepsilon^h_{ij}x^h_i/(1+t_j).$$

Using the symmetry of the Slutsky matrix, the two first order conditions can be rewritten as

$$[(1-c_t)t/(1+t)]\ \{N^A\ [(\varepsilon^A_{NN}+\varepsilon^A_{NS})x^A_N+(\varepsilon^A_{SN}+\varepsilon^A_{SS})x^A_S]+N^B\ [(\varepsilon^B_{NN}+\varepsilon^B_{NS})x^B_N+(\varepsilon^B_{SN}+\varepsilon^B_{SS})x^B_S]\}$$
$$+\ [(1-c_\tau)\tau/(1+\tau)]\ \ N^B(\varepsilon^B_{NI}x^B_N+\varepsilon^B_{SI}x^B_S)$$
$$=-\frac{\lambda(1-c_t)-\beta_N}{\lambda}x_N-\frac{\lambda(1-c_t)-\beta_S}{\lambda}x_S$$

$$[(1-c_t)t/(1+t)]\ N^B(\varepsilon^B_{IN}+\varepsilon^B_{IS})x^B_I+[(1-c_\tau)\tau/(1+\tau)]N^B\varepsilon^B_{II}x^B_I\ =\ -\frac{\lambda(1-c_\tau)-\beta_I}{\lambda}x_I$$

The Slutsky adding up condition (for each type of individual),

$$\sum_j p_j S^k_{ij}=0,\ \ k=A,B,$$

implies that for the taxed goods

$$
\begin{aligned}
&\varepsilon^A_{Nw} + \varepsilon^A_{NE} + \varepsilon^A_{NN} + \varepsilon^A_{NS} = 0 \\
&\varepsilon^A_{Sw} + \varepsilon^A_{SE} + \varepsilon^A_{SN} + \varepsilon^A_{SS} = 0 \\
\\
&\varepsilon^B_{Nw} + \varepsilon^B_{NE} + \varepsilon^B_{NN} + \varepsilon^B_{NS} + \varepsilon^B_{NI} = 0 \\
&\varepsilon^B_{Sw} + \varepsilon^B_{SE} + \varepsilon^B_{SN} + \varepsilon^B_{SS} + \varepsilon^B_{SI} = 0 \\
&\varepsilon^B_{Iw} + \varepsilon^B_{IE} + \varepsilon^B_{IN} + \varepsilon^B_{IS} + \varepsilon^B_{II} = 0
\end{aligned} \tag{4}
$$

Substituting and collecting terms yields, in matrix form,

$$
\begin{Bmatrix}
\begin{matrix} -N^A[x^A_N(\varepsilon^A_{Nw} + \varepsilon^A_{NE}) + x^A_S(\varepsilon^A_{Sw} + \varepsilon^A_{SE})] \\ -N^B[x^B_N(\varepsilon^B_{Nw} + \varepsilon^B_{NE} + \varepsilon^B_{NI}) + x^B_S(\varepsilon^B_{Sw} + \varepsilon^B_{SE} + \varepsilon^B_{SI})] \end{matrix} & N^B(x^B_N \varepsilon^B_{NI} + x^B_S \varepsilon^B_{SI}) \\
-N^B x^B_I(\varepsilon^B_{Iw} + \varepsilon^B_{IE} + \varepsilon^B_{II}) & N^B x^B_I \varepsilon^B_{II}
\end{Bmatrix}
\begin{Bmatrix} (1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau) \end{Bmatrix}
$$
$$
= (-1)\begin{Bmatrix} \dfrac{\lambda(1-c_t)-\beta_N}{\lambda} x_N + \dfrac{\lambda(1-c_t)-\beta_S}{\lambda} x_S \\ \dfrac{\lambda(1-c_\tau)-\beta_I}{\lambda} x_I \end{Bmatrix} \tag{5}
$$

Denote the matrix elements as follows

$$
\begin{Bmatrix} a & b \\ c & d \end{Bmatrix}\begin{Bmatrix} (1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau) \end{Bmatrix} = (-1)\begin{Bmatrix} e \\ f \end{Bmatrix}.
$$

The matrix of the coefficient determinant (D=ad-bc) is positive (because the Slutsky matrix is negative semi-definite), and in general each of the terms in parentheses in the definitions of *e* and *f* is positive (Auerbach and Hines, 2002).[22] Solving for the two tax rate terms and combining yields

---

[22] These conditions are satisfied for all the numerical results reported in Section IV.

$$D\left[\frac{(1-c_t)t}{1+t}-\frac{(1-c_\tau)\tau}{1+\tau}\right]$$

$$= x_I(x_N+x_S)\left\{\begin{array}{l}\left[\frac{\lambda(1-c_t)-\beta_N}{\lambda}\frac{x_N}{x_N+x_S}+\frac{\lambda(1-c_t)-\beta_S}{\lambda}\frac{x_S}{x_N+x_S}\right](\varepsilon_{Iw}^B+\varepsilon_{IE}^B) \\ -\left[\frac{\lambda(1-c_\tau)-\beta_I}{\lambda}\right]\left[\begin{array}{l}\frac{N^A x_N^A}{x_N+x_S}(\varepsilon_{Nw}^A+\varepsilon_{NE}^A)+\frac{N^A x_S^A}{x_N+x_S}(\varepsilon_{Sw}^A+\varepsilon_{SE}^A) \\ +\frac{N^B x_N^B}{x_N+x_S}(\varepsilon_{Nw}^B+\varepsilon_{NE}^B)+\frac{N^B x_S^B}{x_N+x_S}(\varepsilon_{Sw}^B+\varepsilon_{SE}^B)\end{array}\right]\end{array}\right\}$$

Defining the consumption shares of the two goods sold by traditional retailers (noting that the consumer prices of both goods are (1+t)) as

$$f_j^k=\frac{N^k x_j^k}{x_N+x_S}, \quad j=N,S, \quad k=A,B, \tag{6}$$

defining the weighted cross price elasticity of the consumption of the taxed goods with respect to the prices of leisure and the untaxed good as

$$\varepsilon^* = f_N^A(\varepsilon_{Nw}^A+\varepsilon_{NE}^A)+f_S^A(\varepsilon_{Sw}^A+\varepsilon_{SE}^A)+f_N^B(\varepsilon_{Nw}^B+\varepsilon_{NE}^B)+f_S^B(\varepsilon_{Sw}^B+\varepsilon_{SE}^B), \tag{7}$$

and simplifying yields the expression that determines the conditions under which preferential treatment of Internet commerce may be desirable:

$$\frac{\tau}{1+\tau}=\frac{(1-c_t)}{(1-c_\tau)}\frac{t}{1+t} - \frac{x_I(x_N+x_S)}{D(1-c_\tau)}\left\{\begin{array}{l}\left[\frac{\lambda(1-c_t)-\beta_N}{\lambda}\frac{x_N}{x_N+x_S}+\frac{\lambda(1-c_t)-\beta_S}{\lambda}\frac{x_S}{x_N+x_S}\right](\varepsilon_{Iw}^B+\varepsilon_{IE}^B) \\ -\left[\frac{\lambda(1-c_\tau)-\beta_I}{\lambda}\right]\varepsilon^*\end{array}\right\}. \tag{8}$$

This key optimal commodity tax rate expression can be interpreted as follows. Consider first the case in which administrative costs are the same for sales taxation of traditional retailers and Internet sales and the net social marginal valuations of income for the two individuals are identical ($\beta^A=\beta^B=\beta$). Then

$$c_t = c_\tau = c$$
$$\beta_N = \beta_S = \beta_I = \beta .$$

In this case, the optimal tax rate expression becomes

$$\frac{\tau}{1+\tau} = \frac{t}{1+t} - \frac{x_I(x_N + x_S)}{D(1-c)}\left[\frac{(1-c)-\beta}{\lambda}\right][(\varepsilon_{Iw}^B + \varepsilon_{IE}^B) - \varepsilon^*] \ . \qquad (9)$$

This result is the two-person analog to the familiar Corlett-Hague result for the case in which both leisure and one of the consumption goods (x_E) are untaxed. That is, neglecting distributional concerns and differences in administrative costs, if Internet goods are more substitutable for leisure or the untaxed goods, with the elasticities between the taxed goods and leisure and the untaxed goods weighted as shown in the definition of ε* (that is, $(\varepsilon_{Iw}^B + \varepsilon_{IE}^B) - \varepsilon^* > 0$), then the Internet goods should be taxed less (complements with the untaxed good and leisure should be taxed more).[23]

Although it is stated differently, (9) captures the essence of the argument stressed by Goolsbee and Zittrain (based on adapting the Sandmo (1981) labor income taxation model to commodity taxation) that preferential taxation of electronic commerce may be desirable if the own-price elasticity of demand for Internet goods is relatively high for individuals who make purchases over the Internet. Note first that the result on which Goolsbee and Zittrain base their conclusion is a rather weak one. Sandmo (1981) considers optimal linear income taxation in a model with two groups of individuals, only one of which may engage in work in an informal labor market where the tax on labor income is relatively easily avoided and thus can be collected only at a differentially high administrative cost. The model allows differential taxation of labor

---

23 Note that this reasoning suggests that a potential argument for differentially higher taxation of electronic commerce (not considered in the model utilized in this paper) is that the convenience of shopping over the Internet allows leisure time to be used more productively, so that goods purchased over the Internet are more likely to be complementary to leisure.

in the formal and informal markets, once differences in audit probabilities are taken into account. Sandmo shows that the (expected) tax rate on labor in the informal market should be lower than the tax rate in the formal market if the elasticity of labor supply to the informal market with respect to the tax rate in that market is sufficiently large (in absolute value). However, the magnitude of this elasticity must be compared to the cross-price elasticity of labor supply to the formal market with respect to the tax rate in the informal market. That is, preferential tax treatment of wage income earned in the informal sector is desirable only if a tax increase in the informal sector causes a reduction in labor supply to that sector that exceeds in magnitude the associated increase in labor supply to the formal sector. In other words, if taxation of the informal sector merely reallocates labor from the informal to the formal market, then uniform taxation is desirable, regardless of the magnitude of the own-price elasticity of labor supply to the informal sector.[24]

The condition presented above implies a similar result. Consider first the cross-price elasticities involving leisure demand/labor supply effects. Preferential taxation of the Internet goods is desirable if it will have a relatively negative effect on the demand for leisure (Internet goods are substitutes for leisure) and thus a relatively large positive effect on labor supply, which will more than offset the negative effect on labor supply of taxing goods supplied by traditional retailers — that is, only if the net labor supply effect is positive. If instead, the Internet good and goods sold by traditional retailers are equally substitutable for leisure, then uniform taxation is desirable, regardless of the magnitude of the own-price elasticity of demand for the Internet good. For the case analyzed in this paper, this result is augmented by the need to

---

[24] Sandmo also derives a second result, which indicates that under certain conditions the tax rate on labor in the formal sector should be lowered if the elasticity of labor supply to the tax-evading informal market is sufficiently large. In this case, the potential for evasion increases the overall efficiency cost of labor income taxation so that greater reliance on lump sum taxation is desirable in the model. The option of lump sum taxation, however, is assumed to be unavailable in the model utilized in this paper.

consider cross-price effects with the tax-exempt good, where again preferential taxation of the Internet goods is desirable only if it will have a relatively large negative effect on demand for the untaxed good.

This condition can also be related more clearly to the argument made by Goolsbee and Zittrain in an even more specialized case of the model. Suppose that among the three taxed goods (x_N, x_S, and x_I), all cross-price effects are zero except for the cross-price elasticities between the Internet good x_I and x_S (which are large by assumption), and all income effects are zero. In this case,

$$\begin{aligned}
&\varepsilon^A_{Nw} + \varepsilon^A_{NE} = -\varepsilon^A_{NN} \\
&\varepsilon^A_{Sw} + \varepsilon^A_{SE} = -\varepsilon^A_{SS}
\end{aligned}$$

$$\begin{aligned}
&\varepsilon^B_{Nw} + \varepsilon^B_{NE} = -\varepsilon^B_{NN} \\
&\varepsilon^B_{Sw} + \varepsilon^B_{SE} = -(\varepsilon^B_{SS} + \varepsilon^B_{SI}) \\
&\varepsilon^B_{Iw} + \varepsilon^B_{IE} = -(\varepsilon^B_{II} + \varepsilon^B_{IS})
\end{aligned}$$

$$\begin{aligned}
\varepsilon^* &= [f^A_N(-\varepsilon^A_{NN}) + f^B_N(-\varepsilon^B_{NN})] + [f^A_S(-\varepsilon^A_{SS}) + f^B_S(-\varepsilon^B_{SS})] - f^B_S \varepsilon^B_{SI} \\
&= (-\varepsilon_{NN}) + (-\varepsilon_{SS}) - f^B_S \varepsilon^B_{SI}
\end{aligned}$$

Substituting, the optimal tax rate expression (8) becomes

$$\begin{aligned}
\frac{\tau}{1+\tau} &= \frac{t}{1+t} - \frac{x_I(x_N + x_S)}{D(1-c)}[(1-c) - \beta/\lambda][(\varepsilon^B_{Iw} + \varepsilon^B_{IE}) - \varepsilon^*] \\
&= \frac{t}{1+t} - \frac{x_I(x_N + x_S)}{D(1-c)}[(1-c) - \beta/\lambda]\left\{[(-\varepsilon^B_{II}) - \varepsilon^B_{IS}] - [(-\varepsilon_{NN}) + (-\varepsilon_{SS}) - f^B_S \varepsilon^B_{SI}]\right\}
\end{aligned}$$

or

$$\frac{\tau}{1+\tau}-\frac{t}{1+t}=\frac{x_I(x_N+x_S)}{D(1-c)}[(1-c)-\beta/\lambda]\left\{[(-\varepsilon_{NN})+(-\varepsilon_{SS})-f_S^B\varepsilon_{SI}^B]-[(-\varepsilon_{II}^B)-\varepsilon_{IS}^B]\right\}$$
$$=\frac{x_I(x_N+x_S)}{D(1-c)}[(1-c)-\beta/\lambda][(-\varepsilon_{NN})+(-\varepsilon_{SS})-f_S^B\varepsilon_{SI}^B][(-\varepsilon_{II}^B)-\varepsilon_{IS}^B]\left[\frac{1}{(-\varepsilon_{II}^B)-\varepsilon_{IS}^B}-\frac{1}{(-\varepsilon_{NN})+(-\varepsilon_{SS})-f_S^B\varepsilon_{SI}^B}\right] \tag{10}$$

In the absence of any cross price effects, this is simply the Ramsey inverse elasticity rule — the tax rate on the good with the relatively high own price elasticity should have the lower tax rate, as the tax rates, expressed as a percentage of price, are proportional to the inverses of the compensated demand elasticities. This is the point stressed by Goolsbee and Zittrain. That is, if the own price elasticity of the Internet good is large, relative to the own price elasticities of the other taxable goods, optimal tax considerations will tend to imply relatively low tax rates on electronic commerce. However, this result must be tempered by efficiency concerns about distorting consumption between goods sold over the Internet and their substitutes — the standard efficiency concern that if the Internet good is a close substitute for other taxed goods, preferential treatment of the Internet good will cause a large distortion of demand and thus large efficiency losses. This concern is captured by the fact that the own-price effects are tempered by the cross-price effects as indicated above. In addition, as long as the size of the market for Internet goods is relatively small, then

$$\varepsilon_{SI}^B=\varepsilon_{IS}^B\frac{p_I x_I}{p_S x_S}<<\varepsilon_{IS}^B .$$

Thus, it is reasonable to assume that

$$\varepsilon_{IS}^B>>f_S^B\varepsilon_{SI}^B ,$$

and efficiency concerns about distorting consumer decisions regarding purchases from Internet firms rather than traditional retailers work in the direction of increasing the taxation of Internet goods (rather than simply equalizing the two tax rates), thus tempering or reversing any tendency for a tax advantage for Internet goods due to a relative high own price elasticity.[25]

Consider next the role played by distributional concerns. Thus, suppose that the distributional weights (the β's, which include the effects of different individual propensities to consume taxed goods) are different, but the administrative costs of taxing traditional and electronic commerce are still equal. Then the optimal tax rate expression becomes

$$\frac{\tau}{1+\tau}=\frac{t}{1+t} -\frac{x_I(x_N+x_S)}{D(1-c)}\left\{\left[(1-c)-\frac{\beta_N}{\lambda}\frac{x_N}{x_N+x_S}-\frac{\beta_S}{\lambda}\frac{x_S}{x_N+x_S}\right](\varepsilon_{Iw}^B+\varepsilon_{IE}^B)-\left[(1-c)-\frac{\beta_I}{\lambda}\right]\varepsilon^*\right\}$$

or, to isolate the distributional terms,

$$\frac{\tau}{1+\tau}=\frac{t}{1+t} -\frac{x_I(x_N+x_S)}{D(1-c)}[(\varepsilon_{Iw}^B+\varepsilon_{IE}^B)-\varepsilon^*] +\frac{x_I(x_N+x_S)}{D(1-c)}\left[\left(\frac{\beta_N}{\lambda}\frac{x_N}{x_N+x_S}+\frac{\beta_S}{\lambda}\frac{x_S}{x_N+x_S}\right)(\varepsilon_{Iw}^B+\varepsilon_{IE}^B)-\frac{\beta_I}{\lambda}\varepsilon^*\right]. \tag{11}$$

The third term on the right hand side captures the effects of adding distributional concerns. Suppose, consistent with the empirical data described previously, that the Internet good is consumed primarily be relatively wealthy individuals. In that case, β_I will be relatively low in comparison to β_N and β_S.[26] As a result, the third term is more likely to be positive, which will tend to increase the tax rate applied to the Internet good (τ). That is, distributional concerns temper the traditional Corlett-Hague argument expressed in the second term on the right side of

---

25 Note that this argument implies that it is likely that

$$(-\varepsilon_{NN})+(-\varepsilon_{SS})-f_S^B\varepsilon_{SI}^B>0.$$

the equation. Thus, even if Internet goods are substitutes with leisure and/or the untaxed goods and would thus tend to be taxed preferentially, distributional concerns will temper this result and may imply that goods sold over the Internet should be taxed more heavily than goods sold by traditional retailers if the difference in the distributional weights applied to the two individuals is sufficiently large.

Suppose next that distributional concerns are ignored ($\beta^A=\beta^B=\beta$, so that $\beta_N=\beta_S=\beta_I=\beta$), but the administrative costs of taxing Internet goods are greater than those associated with taxing goods sold by traditional retailers ($c_\tau>c_t$).[27] Then the optimal tax rate expression becomes

$$\frac{\tau}{1+\tau}=\frac{(1-c_t)}{(1-c_\tau)}\frac{t}{1+t} \\ -\frac{x_I(x_N+x_S)}{D(1-c_\tau)}\left\{\left[(1-c_t)-\frac{\beta}{\lambda}\right](\varepsilon_{Iw}^B+\varepsilon_{IE}^B)-\left[(1-c_\tau)-\frac{\beta}{\lambda}\right]\varepsilon *\right\}. \tag{12}$$

The first term on the right side of the equation reflects the effects of administrative cost differentials, neglecting the cross price elasticity effects of the taxed goods with the untaxed good and leisure. In this case, if the administrative costs of taxing Internet goods are relatively high ($c_\tau>c_t$), such goods should be taxed at a higher rate (since $(1- c_t)>(1- c_\tau)$). That is, neglecting cross-price effects, uniform taxation would be desirable except for the administrative cost differential, which argues for higher taxes on Internet goods to discourage their consumption and thus lower average administrative costs. Thus, arguments which suggest that preferential taxation of electronic commerce is necessarily desirable because the administrative costs of such taxation are relatively high do not take into account that the lower prices associated

---

26 Note that this characterization assumes that the income effect terms in the definitions of the β's do not dominate the distributional terms.

27 This discussion is similar to the analysis of optimal commodity taxation in the presence of administrative costs in Kaplow (1990).

with lower taxation of electronic commerce will stimulate demand for goods for which tax collection costs are relatively high.

The second term on the right side indicates that the conventional argument that high administrative costs imply that preferential tax treatment is optimal may nevertheless still be valid. This term shows that differential administrative costs change the weighting of the cross-price effects of the taxed goods with the untaxed good and leisure, which tend to reduce (increase) τ if the Internet good is more (less) substitutable with the untaxed good and leisure. When the administrative costs of taxing Internet goods are relatively high ($c_\tau > c_t$), then the weight attached to the term involving the cross-price elasticities of the Internet good with the untaxed good and leisure (the first term in brackets) is relatively large, implying that preferential taxation of Internet goods is more likely to be desirable (as long as $\varepsilon_{Iw}^B + \varepsilon_{IE}^B > 0$). More generally, these results indicate that if the administrative and compliance costs of taxing the Internet good are relatively large, the cross-price effects of this good with leisure and other untaxed goods are even more important in choosing an optimal tax structure.[28]

To sum up, the net results of all these factors, captured in the expression

$$\frac{\tau}{1+\tau} = \frac{(1-c_t)}{(1-c_\tau)}\frac{t}{1+t} - \frac{x_I(x_N + x_S)}{D(1-c_\tau)}\left\{\left[(1-c_t) - \frac{\beta_N}{\lambda}\frac{x_N}{x_N + x_S} - \frac{\beta_S}{\lambda}\frac{x_S}{x_N + x_S}\right](\varepsilon_{Iw}^B + \varepsilon_{IE}^B) - \left[(1-c_\tau) - \frac{\beta_I}{\lambda}\right]\varepsilon^*\right\}. \tag{8}$$

not surprisingly indicate that the case for or against preferential tax treatment of electronic commerce is theoretically ambiguous, with efficiency and administrative cost considerations

[28] Of course, these results depend on the specific form of administrative costs used in the model. For example, if the issue were framed as extending uniform taxation to more goods sold over the Internet and the administrative costs associated with doing so increased more than proportionately, then the extent of base expansion, generally desirable on efficiency grounds, would be limited by increasing marginal administrative costs (Yitzhaki, 1979), On the other hand, the analysis also does not consider the fact that administrative costs tend to increase with the

especially ambiguous while distributional concerns suggest that preferential taxation is not desirable. The following section attempts to shed some additional light on this issue, by performing some calculations of this optimal tax condition for various parameter values.

IV. SOME ILLUSTRATIVE EXAMPLES

As is inevitably the case for optimal tax problems, the final results are theoretically ambiguous and depend on the values of various key parameters. This section presents some sample calculations for the tax rates t and τ, using the optimal tax condition (8) and a balanced budget requirement under which the combined revenues from any configuration of taxation of traditional and electronic commerce equals the revenues, net of administrative costs, in the initial equilibrium. The measure of preferential treatment for Internet goods is defined as the tax differential favoring electronic commerce, as a percentage of the tax rate applied to traditional commerce, or

$$\phi = \frac{t-\tau}{t}.$$

Expressed in percentage terms, if ϕ=100 the optimal tax structure implies tax exemption for electronic commerce, uniform treatment is reflected as ϕ=0, and ϕ<0 (ϕ>100) implies that electronic commerce should be taxed more heavily than traditional commerce (subsidized).

The calculations are based on parameter values taken from the empirical literature when available and plausible parameter values when empirical data are not available. The analysis uses a partial equilibrium approach that ignores general equilibrium reactions between changes in the tax rates and various parameter values (such as income shares, income and price elasticities, the marginal efficiency cost of taxation and, for purposes of the government budget

---

existence of rate differentials, as goods must be allocated to different tax classes. If this factor were considered, constraint, the sizes of the sales tax bases). It should thus be viewed as only suggestive of the optimal tax differentials that might arise in a more complete general equilibrium analysis. Given the considerable uncertainty surrounding the parameter values used, the calculations are performed for a range of parameter values.

PARAMETER VALUES

The parameter values used in the calculations were obtained as follows. In its study *A Nation Online: How Americans are Expanding Their Use of the Internet*, the U.S. Department of Commerce (2002) reports that in September 2001, 53.9 percent of the population (of age 3 and over) were using the Internet. Among Internet users, 39 percent of individuals are making online purchases. Thus approximately 21% of the population are Internet shoppers (.21=.39x.539), and 79% are not. Accordingly, the calculations assume $N^A/N=0.79$ and $N^B/N=0.21$. Assuming that these ratios can be applied directly to households (they are calculated for individuals) and using the 2001 figure for a population of 109.3 million households (Census Bureau, 2001), this implies N^A=86.3 million and N^B =23.0 million.

Goolsbee and Zittrain (1999) provide information, based on 1997 data collected by Forrester Research, grouped by individuals who use and do not use the Internet. In particular, they report that the average income of individuals who do not use the Internet was $35,600, while the average income of Internet users was $57,200. Converting these figures to 2001 dollars yields values of $39,300 and $63,100. The ratio of the latter to the former is 1.6. Since data presented in U.S. Department of Commerce (2002) indicate that use by lower income households has increased more rapidly in the intervening years (and since the

---

there would be an additional tendency toward uniform rates.

consumption/income ratio declines with income), the assumed ratio of consumption expenditures of group B to group A is adjusted downward slightly to 1.5.[29]

With total personal consumption expenditures, exclusive of sales taxes paid, of $7,045 billion in 2001 (U.S Department of Commerce, Bureau of Economic Analysis, 2002), these assumptions imply that the total consumption levels of groups A and B are $5,037 billion and $2,008 billion. This consumption must be allocated among the four different goods in the model. Given the degree of aggregation in these data, any such allocation will of course be quite arbitrary; the definitions are chosen so that the amount of personal expenditures that could potential be made over the Internet is fairly broad. Using the consumption expenditures classifications used in the National Income and Product Accounts, exempt goods (X_E) are defined as food, housing, transportation and medical care, goods that are not substitutable for goods purchased over the Internet (X_N) as defined as motor vehicles[30] and parts, energy goods and services, recreation, other services[31] and other household operation items, and goods that are substitutable for goods purchased over the Internet or are actually purchased over the Internet (X_S and X_I) are defined as furniture and other household equipment, clothing and shoes, and all other durable and non-durable goods. The latter category must be divided between X_S and X_I. Although Internet sales, especially of consumer goods, are still relatively small (Cline and Neubig, 1999), they are growing rapidly. Since the projections of Bruce and Fox (2000) suggest

---

[29] Note that these figures are used as the consumption of individuals of type A and type B. This assumes that consumption is proportional to income for the two groups, and that the income levels of Internet shoppers are the same as those of Internet users. Goolsbee and Zittrain (1999) show that the variation in Internet usage across income groups is far greater than the variation in purchasing over the Internet, given Internet usage (although their results indicate that high income Internet users are somewhat more likely to be Internet shoppers).

[30] Although there is a small trade in motor vehicles over the Internet, such purchases are typically fully subject to either sales tax or a similar motor vehicle tax when the vehicle is registered. Thus, since most motor vehicle are not purchased over the Internet and all motor vehicles are subject to sales or motor vehicle taxes, motor vehicles are included in the group of goods that are not substitutable for goods purchased over the Internet and fully taxed at rate the standard sales tax rate t.

[31] Since most states are increasing the extent to which they tax consumer services (Hendrix and Zodrow, 2003), consumer services other than housing, transportation and medical care as treated as taxed goods, although this categorization is obviously fairly arbitrary.

potentially taxable sales of consumer goods over the Internet in 2003 of about $130 billion (slightly under 10 percent of all consumer expenditures in categories X_S and X_I), this figure is used in allocating goods between the two groups.[32] Bruce and Fox (2000) indicate that the average US state sales tax rate (t) is 6.38 percent, and that about 21 percent of potentially taxable consumer sales over the Internet are taxed, either because the selling firm has nexus in the consumer's state of residence or because the state collects use tax, implying an effective tax rate on such sales (τ) of 1.33 percent. The implied level of sales tax revenue, given the division of goods specified above, is $223.3 billion, yielding total gross consumer expenditures of $7,269 billion.[33]

Expenditures on consumer goods are allocated between the two groups A and B using data from Jorgenson (1997), which provides expenditure shares and income elasticities by income level for the classification of goods in the National Income and Product Accounts; these data are interpolated to fit the classifications used for the two expenditure groups, including sales taxes paid, and for the two income levels used in the analysis.[34] The results of this interpolation are presented in Table 1. The resulting expenditure shares are fairly similar, with the lower income individuals in Group A spending a somewhat larger fraction of their income on exempt goods and the higher income individuals in Group B spending a somewhat larger share of income on goods that can be purchased over the Internet and their substitutes. The income elasticities range from 0.8-1.3, with the lower income individuals in Group A having a higher

---

32 Although this estimate, which is adjusted to reflect the larger values in unpublished data cited by Bruce and Fox (2000), is on the high end of existing estimates, especially when compared with Cline and Neubig (1999), it is used to increase the likelihood that tax treatment of electronic commerce will be desirable in the optimal tax framework.

33 This figure is not directly comparable to actual revenues, since the tax bases in this analysis are so approximate and sales taxation of business purchases is ignored; nevertheless, this figure is very close to the actual level of general sales tax revenues in 2001 (U.S. Census Bureau, 2004-05).

34 Although these data are quite dated (1972), they provide the only information on expenditures shares and income elasticities by income and by commodity group that I was able to find.

income elasticity for exempt goods and individuals in Group B having a higher income elasticity for goods substitutable with goods that can be purchased over the Internet.

Table 1. Expenditure Shares and Income Elasticities

Consumption Good	Expenditure Share, Group A	Expenditure Share, Group B	Income Elasticity Group A	Income Elasticity Group B
Good E	0.495	0.421	0.909	0.834
Good N	0.334	0.326	1.026	1.015
Good S	0.171	0.189	1.121	1.258
Good I	0	0.064	0	1.258

The figures on expenditure shares by type of individual can then be used to calculate the distribution of the total consumption of the two groups across the four goods. The results of this calculation are presented in Table 2. Note that by assumption only individuals in Group B have expenditures on consumption of good X_I.

Table 2. Consumption Expenditures by Group

Consumption Good	Total Expenditures ($ billion)	Share of Total Consumption	Expenditures of Group A ($ billion)	Expenditures of Group B ($ billion)
Good E	3,442	0.474	2,569	873
Good N	2,412	0.332	1,735	677
Good S	1,283	0.176	890	392
Good I	132	0.018	0	132
Total	7,269	1.000	5,194	2,074

The remaining parameters needed for the calculation of the optimality condition (8) are the various own-price and cross-price elasticities. Only limited evidence is available on these, so in many cases calculations are performed for a range of plausible values. As noted above, the results presented in Goolsbee (2000) suggest a very high own-price elasticity of Internet goods – in the context of the model utilized in this paper, the estimated reduction in Internet purchases due to the taxation of electronic commerce implies an average own-price demand elasticity roughly on the order of $\varepsilon_{II}^{B} = -4.0$. By comparison, Alm and Melnik (2003), using a more recent and comprehensive data set, present results that suggest a much lower own-price demand elasticity for Internet goods, on the order of one-fourth that estimated by Goolsbee (2000). However, since the implications of a high own-price demand elasticity for Internet goods are the focus of this paper – and in the spirit of making assumptions which tend to make the strongest case for preferential treatment of electronic commerce – the analysis will assume the higher value in the simulations presented below.

Another key parameter is the cross-price elasticity of the various goods with leisure and the tax-exempt good. Fullerton and Rogers (1993) use an uncompensated elasticity of substitution between consumption and leisure of 0.50 in their lifetime tax incidence general equilibrium model which, with the CES utility function they employ and a time endowment equally split between consumption and non-market activity, implies a compensated cross-price elasticity of demand with leisure of 0.25. Accordingly, 0.25 is used as the benchmark value for the compensated cross-price elasticities of the various goods with leisure. Since the tax-exempt good consists primarily of basic necessities, all cross-price elasticities involving the tax-exempt good are assumed to be zero. Finally, the cross-price elasticity of the goods that are assumed to be not substitutable with the goods sold over the Internet is also assumed to be zero.

For administration and compliance costs, Slemrod and Bakija (1996) report from a sample of eight states that administrative costs under the state sales tax were between 0.4-1.0 percent of revenues and compliance costs were between 2-3.8 percent in seven of those states. Accordingly, the analysis assumes that marginal costs are constant with c_t=0.35, which is near the midpoint of this range, and that c_τ is some multiple of c_t.

Consider next the social welfare optimization problem. The analysis assumes that the individual utility functions, which are characterized by heterogeneous preferences, are always cardinalized so that the marginal utility of income for both types of individuals is one; this implies that differences in individual preferences do not by themselves (in the absence of any social concerns about the distribution of income) give rise tax differentials.[35] The analysis

[35] See Boadway, Marchand, Pestieau and del Mar Racionero (2002) for a discussion of such cardinalization for social welfare optimization with individuals with heterogeneous preferences.

initially assumes no social weighting ($\gamma^A = \gamma^B = 1$), but also considers various weighting schemes that place a lower weight γ^B on the relatively higher income individuals. For the calculation of λ, the marginal social cost of raising revenue, the marginal excess burden of the system of sales taxes is assumed to be constant at 10 percent (recall that endogenous variations in this value due to changes in the tax rates applied to traditional and electronic commerce are ignored in the partial equilibrium analysis utilized in this paper) and administrative and compliance costs are as described above. This figure is roughly consistent with general equilibrium results on the efficiency costs of a national value-added tax presented by Ballard, Scholz and Shoven (1987), adjusted upward to reflect variation in sales tax rates across the states and the fact that much of the sales tax in the U.S. functions as a distortionary cascading tax on business inputs.[36] Finally, the various β values (2-3) are calculated using these values and the expenditure shares and income elasticities presented in Table 1.

RESULTS

The first set of results establishes a benchmark case, in which distributional concerns are ignored, administrative and compliance costs for taxing Internet goods are assumed to equal those for traditional commerce, and all compensated cross-price elasticities with respect to leisure and the untaxed goods are set equal to 0.25. In this benchmark case, which is designed to maximize the optimality of equal taxation, the optimal tax rate on Internet goods is 6.09 percent, slightly less than the tax rate on traditional goods of 6.20 percent. This slight differential reflects the fact that even though $\gamma^A=\gamma^B=1$, $\beta^B/\lambda=0.946$ is slightly larger than $\beta^A/\lambda=0.940$ because individuals of type B are slightly more likely to spend additional income on

[36] Recall that Ring (1999) estimates that 60 percent of the state sales tax base consists of consumer purchases. The resulting cascading of tax burden implies that the US state sales the tax system is considerably more inefficient than

taxed goods. As a result, slightly preferential tax treatment of Internet goods is desirable because it provides more income to individuals of type B, who in turn spend it disproportionately on taxed goods. The resulting "optimal" tax differential of 0.11 percentage points (or a value of ϕ=1.8 percent) due to this factor is, however, negligible.

Consider next variations in the degree of substitutability of Internet goods for leisure (ε_{IW}^{B}).[37] Larger (smaller) values of this elasticity should lead to lower (higher) tax rates on Internet goods, as they are less (more) complementary with leisure, and this pattern is observed in Table 3. However, variations in the cross-price elasticity over a considerable range ($-0.50 < \varepsilon_{IW}^{B} < 1.0$) lead to only moderate variation in the tax rates, as the optimal tax differentials vary from -0.66 to 0.88 percentage points, with the e-commerce tax preference measure (ϕ) clustering in the neighborhood of zero ($-10.6\% < \phi < 14.2\%$), indicating that the optimal tax preference for Internet goods is at most about 14 percent for the range of parameter values analyzed, and should in fact be negative for the values of $\varepsilon_{Iw}^{B} < 0.25$, reflecting a surtax as large as nearly 11 percent.

---

the truly consumption-based VAT studied by Ballard, Scholz and Shoven (1987).

[37] All variations in the relative substitutability of the Internet good for leisure (or for the untaxed good) are assumed to be accompanied by identical variations for the closely substitutable good X_S.

Table 3. Variations in Substitutability of X_I for Leisure

ε_{IW}^{B}	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
t	6.17%	6.18%	6.19%	6.20%	6.21%	6.22%	6.23%
τ	6.83%	6.58%	6.34%	6.09%	5.84%	5.60%	5.35%
t-τ	-0.66%	-0.40%	-0.15%	0.11%	0.37%	0.62%	0.88%
ϕ	-10.6%	-6.5%	-2.4%	1.8%	5.9%	10.0%	14.2%

The optimality condition (8) also indicates that the cross-price elasticity of the Internet good with untaxed goods is also an important parameter in determining the extent of taxation of electronic commerce. Although the base case assumption that this elasticity is zero seems plausible, a higher (lower) values of ε_{IE}^{B}, for any given value of ε_{Iw}^{B} ,would tend to imply lower (higher) taxation of the Internet good, as it would be less (more) complementary with the untaxed good. This is demonstrated in Table 4, which gives the optimal tax differentials for the same range of ε_{Iw}^{B}, that is, $-0.50 < \varepsilon_{IE}^{B} < 1.00$, when $\varepsilon_{IE}^{B} = 0.25$ rather that the benchmark value of $\varepsilon_{IE}^{B} = 0$. As expected, this increases the likelihood that preferential taxation of electronic commerce is desirable, although the effects are fairly modest. Specifically, the optimal tax differentials now vary from -0.40 to 1.14 percentage points, with the e-commerce tax preference measure (ϕ) shifting up about 4 percentage points, as it varies from ($-6.5\% < \phi < 18.3\%$). Thus, in this case, the maximum optimal tax preference for Internet goods increases from around 14 percent to roughly 18 percent for the range of parameter values analyzed, and is negative only for the negative values of ε_{Iw}^{B}, with a maximum surtax of 6.5 percent.

Table 4. Variations in Substitutability of X_I for Leisure ($\varepsilon_{IE}^{B} = 0.25$)

ε_{IW}^{B}	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
t	6.18%	6.19%	6.20%	6.21%	6.22%	6.23%	6.24%
τ	6.58%	6.34%	6.09%	5.84%	5.60%	5.35%	5.10%
t-τ	-0.40%	-0.15%	0.11%	0.37%	0.62%	0.88%	1.14%
ϕ	-6.5%	-2.4%	1.8%	5.9%	10.0%	14.2%	18.3%

The effect of adding distributional concerns to the analysis is less ambiguous and somewhat more significant. Suppose that social concern about the (current) fact that Internet users are relatively high income individuals is expressed by utilizing weights in the social welfare function such that $\gamma^{B} < \gamma^{A} = 1$. Although the appropriate weights, if any, are inherently subjective, a value of γ^{B} outside the range of $0.5 < \gamma^{B} < 1$ seems implausible, especially since the consumption levels of the two groups are not radically different (recall that the consumption level of group B is 1.5 times the consumption of group A).[38] Table 5 provides results for this variation in γ^{B} for the benchmark case in which $\varepsilon_{IW}^{B} = 0.25$ (and $\varepsilon_{IE}^{B} = 0$). These results demonstrate that such weighting significantly affects the optimal tax differential, implying a surtax on the Internet good – since it is consumed disproportionately by the relatively high income group – that goes as high as 2 percentage points. Similarly, the e-commerce tax preference measure ϕ is negative for all values of $\gamma^{B} < 1$, implying surtaxes that range up to 33 percent.

Table 5. Variations in Social Weight on Higher Income Individuals

($\gamma^A = 1.0,\ \varepsilon^B_{IW} = 0.25,\ \varepsilon^B_{IE} = 0$ **)**

γ^B	1.00	0.95	0.90	0.80	0.70	0.60	0.50
t	6.20%	6.18%	6.17%	6.15%	6.14%	6.13%	6.12%
τ	6.09%	6.67%	7.06%	7.55%	7.84%	8.04%	8.18%
t-τ	0.11%	-0.49%	-0.90%	-1.40%	-1.71%	-1.91%	-2.05%
ϕ	1.8%	-8.0%	-14.5%	-22.8%	-27.8%	-31.1%	-33.5%

Differential administrative and compliance costs also play an important role in determining the optimal tax rate on Internet goods. For example, consider the benchmark case ($\varepsilon^B_{IW} = 0.25, \varepsilon^B_{IE} = 0,\ \gamma^A = \gamma^B = 1.0$), and suppose that the marginal administrative and compliance costs of taxing Internet goods are somewhere between equal to and twice the costs for other goods of 3.5 percent of revenues ($c_t = 0.035$). Table 6 presents the results for this case.

[38] The maximum differential in the distributional weights shown in Table 5 corresponds roughly to a social welfare function with an elasticity of the marginal social valuation of individual utility of one, as a fifty percent increase in an individual's consumption gives rise to a fifty percent reduction in the individual's social weight.

Table 6. Variations in Administrative and Compliance Costs for X_I

($c_t = 0.035,\ \varepsilon_{IW}^{B} = 0.25, \varepsilon_{IE}^{B} = 0,\ \gamma^{A} = \gamma^{B} = 1.0$ **)**

c_τ	0.035	0.040	0.045	0.050	0.055	0.060	0.070
t	6.20%	6.21%	6.21%	6.21%	6.22%	6.22%	6.23%
τ	6.09%	6.01%	5.93%	5.85%	5.76%	5.68%	5.50%
t-τ	0.11%	0.19%	0.28%	0.37%	0.45%	0.54%	0.73%
ϕ	1.8%	3.1%	4.5%	5.9%	7.3%	8.7%	11.7%

These results suggest that differentially high administrative and compliance costs for taxing Internet goods put moderate downward pressure on τ. In particular, although there is a tendency for higher administrative and compliance costs to lead to higher tax rates on Internet goods in order to discourage consumption of goods with high tax administration costs, this effect is outweighed by the interaction between the higher administrative and compliance costs and the cross-price elasticity effects in (8), as described in Section III. The net result is that the optimal sales tax differential favoring Internet goods increases monotonically, ranging from $0.11 - 0.73$ percentage points, with the Internet good preference parameter ranging from $1.8\% < \phi < 11.7\%$. Nevertheless, given the significant increase in costs at the high end of this spectrum (a doubling from 3.5 to 7.0 percent of total revenues), the fact that the optimal tax preference for Internet goods is a maximum of less than 12 percent suggests that higher administrative and compliance costs by themselves do not justify a significant preference for electronic commerce.

A wide variety of interactions among the various parameters can also be examined. Consider first the interactions between variations in administrative and compliance costs and differences in the cross-price elasticities of the Internet good with leisure and untaxed goods. These are explored in Table 7 which examines the effects of higher administrative and compliance costs for the case in which the Internet good relatively substitutable with leisure and the untaxed good ($\varepsilon_{IW}^{B} = 0.75, \varepsilon_{IE}^{B} = 0.25$), and in Table 8 which illustrates the case in which the Internet good is relatively complementary for these goods ($\varepsilon_{IW}^{B} = -0.25, \varepsilon_{IE}^{B} = -0.25$). These results indicate that the differences in cross-price elasticities have the expected effect on the optimal tax rates on Internet goods. When Internet goods are relatively substitutable with leisure and the untaxed good, preferential taxation of Internet goods is always desirable, with differentials that range from $0.88 - 1.8$ percentage points (Table 7). Even in this case, however, the maximum value of the Internet good tax preference parameter ϕ for the range of parameters considered is still less than 30 percent. In contrast, when Internet goods are relatively complementary with both leisure and the untaxed good, the effects of increases in administrative costs are never strong enough to offset the tendency to tax leisure complements at differentially higher rates, with surtaxes on Internet goods ranging from 10.6 to 5.7 percent (Table 8).

Table 7. Variations in Administrative and Compliance Costs for X_I

($c_t = 0.035,\ \varepsilon_{IW}^B = 0.75,\ \varepsilon_{IE}^B = 0.25,\ \gamma^A = \gamma^B = 1.0$)

c_τ	0.035	0.040	0.045	0.050	0.055	0.060	0.070
t	6.23%	6.24%	6.24%	6.25%	6.25%	6.26%	6.27%
τ	5.35%	5.23%	5.10%	4.98%	4.85%	4.73%	4.46%
t-τ	0.88%	1.01%	1.14%	1.27%	1.40%	1.53%	1.81%
ϕ	14.2%	16.2%	18.2%	20.3%	22.4%	24.5%	28.8%

Table 8. Variations in Administrative and Compliance Costs for X_I

($c_t = 0.035,\ \varepsilon_{IW}^B = -0.25,\ \varepsilon_{IE}^B = -0.25,\ \gamma^A = \gamma^B = 1.0$)

c_τ	0.035	0.040	0.045	0.050	0.055	0.060	0.070
t	6.17%	6.18%	6.18%	6.18%	6.19%	6.19%	6.19%
τ	6.83%	6.79%	6.75%	6.71%	6.67%	6.63%	6.55%
t-τ	-0.66%	-0.62%	-0.57%	-0.53%	-0.49%	-0.45%	-0.35%
ϕ	-10.6%	-10.0%	-9.3%	-8.6%	-7.9%	-7.2%	-5.7%

Finally, the interaction between distributional concerns and differential administrative costs is examined in Tables 9 and 10. Consider first the benchmark case ($\varepsilon_{IW}^B = 0.25,\ \varepsilon_{IE}^B = 0$) but suppose that administrative and compliance costs for taxing Internet goods are twice those for other goods ($c_t = 0.035,\ c_\tau = 0.070$), and the range of distributional weights is the same as in Table 5. The results for this case are given in Table 9, and suggest that distributional concerns rapidly outweigh the effects of differential administrative costs, as the optimal tax rate on

Internet goods roughly equals the rate on other goods for $\gamma^B = 0.95$ (ϕ=−1.3%) and the optimal tax preference parameter for the Internet good indicates a surtax of nearly 35 percent when $\gamma^B = 0.5$.

Table 9. Variations in Social Weight on Higher Income Individuals

($\gamma^A = 1.0$**,** $c_t = 0.035$, $c_\tau = 0.070$, $\varepsilon^B_{IW} = 0.25$, $\varepsilon^B_{IE} = 0$ **)**

γ^B	1.00	0.95	0.90	0.80	0.70	0.60	0.50
t	6.23%	6.20%	6.18%	6.16%	6.15%	6.14%	6.13%
τ	5.50%	6.28%	6.80%	7.45%	7.84%	8.10%	8.28%
t-τ	0.73%	-0.08%	-0.62%	-1.29%	-1.69%	-1.96%	-2.15%
ϕ	11.7%	-1.3%	-10.0%	-20.9%	-27.5%	-31.9%	-35.1%

This strong result is only tempered somewhat if Internet goods are relatively substitutable for leisure and untaxed goods. To take the case among those analyzed thus far most favorable to a tax preference for electronic commerce, suppose that $\varepsilon^B_{IW} = 0.75, \varepsilon^B_{IE} = 0.25$ and administrative costs for taxing Internet goods are still double those of other goods ($c_t = 0.035$, $c_\tau = 0.070$). In this case, even though the tax preference parameter is nearly thirty percent (ϕ=28.8%) in the absence of distributional concerns, such concerns are sufficiently important to result in a roughly uniform taxation for $\gamma^B = 0.9$ ($\phi = 0.4\%$), and the tax preference parameter indicates a surtax on Internet goods of more than 33 percent when $\gamma^B = 0.5$.

Table 10. Variations in Social Weight on Higher Income Individuals

($\gamma^A = 1.0$, $c_t = 0.035$, $c_\tau = 0.070$, $\varepsilon^B_{IW} = 0.75$, $\varepsilon^B_{IE} = 0.25$ **)**

γ^B	1.00	0.95	0.90	0.80	0.70	0.60	0.50
t	6.27%	6.23%	6.21%	6.18%	6.16%	6.14%	6.14%
τ	4.46%	5.49%	6.18%	7.05%	7.57%	7.92%	8.17%
t-τ	1.81%	0.74%	0.03%	-0.87%	-1.41%	-1.78%	-2.04%
φ	28.8%	11.9%	0.4%	-14.1%	-23.0%	-28.9%	-33.2%

V. CONCLUSION

The goal of this research was to identify the factors that determine whether preferential taxation of electronic commerce is desirable from an optimal taxation perspective and provide some calculations that illustrate the implications of plausible values for these factors. The model constructed built on existing results from the literatures on optimal commodity taxation including both efficiency and equity concerns, taxation and administrative costs, and the taxation of electronic commerce. The analysis stressed a balancing of the tendency toward uniform taxation due to efficiency concerns about distorting consumer decisions across consumption commodities as well as the essential physical similarity of goods sold over the Internet and by traditional retailers against the tendency for differential taxation attributable to differences between the goods, including differences in various demand elasticities, distributional preferences and administrative and compliance costs. The analysis identified the parameters that are critical in characterizing these offsetting tendencies, and provided some illustrative calculations as a means of determining their net effects and thus ascertaining whether it appears that uniform or preferential taxation of electronic commerce is desirable.

The results presented in this paper indicate that, as is generally the case in optimal taxation problems, the optimal tax treatment of traditional and electronic commerce varies considerably depending on key parameter values (highlighting the need for more empirical data on these parameters), implying preferential treatment of electronic commerce in some cases and higher taxes in others. Nevertheless, even though the assumptions of the model were generally chosen to maximize the likelihood that preferential treatment of electronic commerce would be desirable, the results suggest that, in the practical terms of the current debate, uniform tax treatment of traditional and electronic commerce is much more likely to be desirable than tax exemption (or, alternatively, a significant tax preference for e-commerce). In none of the cases analyzed did the optimal tax rate approach zero, with the largest "optimal" tax preference for electronic commerce on the order of 30 percent, and many much less than that. A key factor leading to prescriptions for moderate preferential treatment of sales over the Internet was relatively high administrative and compliance costs, lending some credence to the argument that uniform taxation is much more likely to be desirable if advances in computer software or simplification efforts such as those proposed and to some extent implemented under the Streamlined Sales Tax Project (or even more radical proposals for simplification such as that advanced by McLure (2000)) are successful in reducing the costs of administering and complying with taxation of sales over the Internet. On the other hand, differentially higher taxation of Internet goods was also optimal in a variety of cases, with the always controversial addition of distributional concerns playing a large role in prescribing relatively high tax rates on Internet goods, and in some cases resulting in surtaxes in the neighborhood of 35 percent. Indeed, the simplest characterization of the results presented in this paper is that the optimal tax rates on electronic commerce tend to cluster around the rate applied to traditional commerce. Thus, on balance, it seems clear that tax exemption of Internet goods is very unlikely to be even close to optimal, and the optimal differentials calculated in this paper suggest that it is unlikely that the traditional prescription of uniform taxation should be over-ridden by optimal taxation concerns, especially once the administrative and political difficulties of implementing differential taxation are taken into account.

REFERENCES

Advisory Commission on Electronic Commerce, 2000. "Report to Congress." <http://www.ecommercecommission.org/acec_report.pdf>

Alm, James and Mikhail I. Melnick, 2003. "Sales Taxes and the Decision to Purchase Online," manuscript, Georgia State University.

Auerbach, Alan J. and James R. Hines, 2002. "Taxation and Economic Efficiency," in Alan J. Auerbach and Martin Feldstein (eds.), *Handbook of Public Economics, Volume 3* (New York: Elsevier).

Ballard, Charles L., John Karl Scholz and John B. Shoven, 1987. "The Value-added Tax: A General Equilibrium Look at Its Efficiency and Incidence," in Martin Feldstein (ed.), *The Effects of Taxation on Capital Accumulation.* Chicago: University of Chicago Press.

Boadway, Robin, Maurice Marchand, Pierre Pestieau and Maria del Mar Racionero, 2002. "Optimal Redistribution with Heterogeneous Preferences for Leisure," *Journal of Public Economic Theory* 4: 475-498.

Bruce, Donald, John Deskins and William F. Fox, 2004. "Has Internet Access Taxation Affected Internet Use?" *Public Finance Review* 32: 131-147.

Bruce, Donald, and William F. Fox, 2000. "E-Commerce in the Context of Declining State Sales Tax Bases, *National Tax Journal* 53: 1373-1388.

Bruce, Donald, William F. Fox and Matthew Murray, 2003. "To Tax or Not To Tax? The Case of Electronic Commerce," *Contemporary Economic Policy* 21: 25-40.

Cline, Robert J. and Thomas S. Neubig, 1999. "The Sky is Not Falling: Why State and Local Revenues Were Not Significantly Affected in 1998," *State Tax Notes* 43.

Corlett, W. and D. Hague, 1953. "Complementarity and the Excess Burden of Taxation," *Review of Economic Studies* 21: 21-30.

Fox, William F. and Matthew Murray, 1997. "The Sales Tax and Electronic Commerce: So What's New?" *National Tax Journal* 50: 573 –592.

Fox, William F. and LeAnn Luna, 2003. "Taxing E-Commerce: Neutral Taxation if Best for the Industry and the Economy." Manuscript, University of Tennessee at Knoxville.

Goolsbee Austan, 2000a. "In a World Without Borders: The Impact of Taxes on Internet Commerce," *Quarterly Journal of Economics* 115: 561-576.

Goolsbee, Austan, 2000b. "Internet Commerce, Tax Sensitivity and the Generation Gap," in James Poterba, ed., *Tax Policy and the Economy*, vol. 14 (Cambridge MA: MIT Press).

Goolsbee Austan and Jonathan Zittrain, 1999. "Evaluating the Costs and Benefits of Taxing Internet Commerce," *National Tax Journal* 52: 413-428.

Fullerton, Don and Diane Lim Rogers, 1993. *Who Bears the Lifetime Tax Burden?* Washington DC: The Brookings Institution.

Harberger, Arnold, 1990. "The Uniform Tax Controversy," in Vito Tanzi (ed.), *Public Finance, Trade and Development.* Detroit MI: Wayne State University Press.

Heller, Walter P. and Karl Shell, 1971. "On Optimal Taxation with Costly Administration," *American Economic Review* 64: 338-345.

Hendrix, Michele and George R. Zodrow, 2003. "Sales Taxation of Services: An Economic Analysis," *Florida State Law Review*, 30: 411-433.

Jorgenson, Dale J., 1997. *Welfare, Volume 1: Aggregate Consumer Behavior*. Cambridge MA: MIT Press.

Kaplow, Louis, 1990. "Optimal Taxation with Costly Enforcement and Evasion," *Journal of Public Economics* 43: 221-236.

McLure, Charles E., Jr., 2000. "Rethinking State and Local Reliance on the Retail Sales Tax: Should We Fix the State Sales Tax or Discard It?" *Brigham Young University Law Review*, pp. 77-137.

McLure, Charles E., Jr., 2002. "Taxing Electronic Commerce: Legal, Economic, Administrative, and Political Issues," *Urban Lawyer*, 34, 487-520.

McLure, Charles E., Jr. and Walter Hellerstein, 2004. "Congressional Intervention in State Taxation: A Normative Analysis of Three Proposals," *State Tax Notes* 31 (March 1): 721-735.

Mikesell, John L., 2000. "Remote Vendors and American Sales and Use Taxation: The Balance Between Fixing the Problem and Fixing the Tax," *National Tax Journal* 53: 1273-1286.

National Tax Association, 1999. "Communications and Electronic Commerce Tax Project: Final Report." <http://www.ntanet.org/ecommerce/final.pdf>

Ring, Raymond, 1999. "Consumers' Share and Producers' Share of the General Sales Tax." *National Tax Journal* 52: 79-90.

Saez, Emmanuel, 2000. "The Desirability of Commodity Taxation under Non-Linear Income Taxation and Heterogeneous Tastes," *National Bureau of Economic Research Working Paper W8029.*

Saez, Emmanuel, 2002. "Direct or Indirect Tax Instruments for Redistribution: Short-run versus Long-run," *National Bureau of Economic Research Working Paper W8833.*

Slemrod, Joel, 1990. "Optimal Taxation and Optimal Tax Systems," *Journal of Economic Perspectives* 4: 157-178.

Slemrod, Joel and Jon Bakija, 1996. *Taxing Ourselves: A Citizen's Guide to the Great Debate over Tax Reform.* Cambridge MA: MIT Press.

Streamlined Sales Tax Project, 2001. "Streamlined Sales Tax System for the 21st Century." <http://www.geocities.com/streamlined2000/>

U. S. Bureau of the Census, 2004-2005. *Statistical Abstract of the United States: 2004-2005.* Washington DC: U. S. Bureau of the Census.

U.S. Department of Commerce, 2002. *A Nation Online: How Americans are Expanding Their Use of the Internet.* Washington DC: U.S. Department of Commerce.

Yitzhaki, Shlomo, 1979. "A Note on Optimal Taxation and Administrative Costs," *American Economic Review* 69: 475-480.

Zodrow, George R., 1999. *State Sales and Income Taxes.* College Station: Texas A&M Press.

Zodrow, George R., 2000. "Evaluating the Case for a Temporary Moratorium on the Taxation of Electronic Commerce," *State Tax Notes*, 19 (December 4), pp. 1515-1521.

Zodrow, George R., 2002. "Network Externalities and Indirect Tax Preferences for Electronic Commerce," *International Tax and Public Finance*10: 79-97.

Zodrow, George R. and Michele E. Hendrix, 2003. "Sales Taxation of Services: An Economic Analysis," *Florida State Law Review* 30 (Spring): 411-433.

APPENDIX

The two first order conditions for the tax rates t and τ presented in the text are

$$\begin{Bmatrix} N^A[x_N^A(\varepsilon_{Nw}^A+\varepsilon_{NE}^A)+x_S^A(\varepsilon_{Sw}^A+\varepsilon_{SE}^A)] + N^B[x_N^B(\varepsilon_{Nw}^B+\varepsilon_{NE}^B+\varepsilon_{NI}^B)+x_S^B(\varepsilon_{Sw}^B+\varepsilon_{SE}^B+\varepsilon_{SI}^B)] & -N^B(x_N^B\varepsilon_{NI}^B+x_S^B\varepsilon_{SI}^B) \\ N^B x_I^B(\varepsilon_{Iw}^B+\varepsilon_{IE}^B+\varepsilon_{II}^B) & -N^B x_I^B\varepsilon_{II}^B \end{Bmatrix} \begin{Bmatrix} (1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau) \end{Bmatrix}$$

$$= \begin{Bmatrix} [(1-c_t)-\beta_N/\lambda]\,x_N + [(1-c_t)-\beta_s/\lambda]\,x_S \\ [(1-c_\tau)-\beta_i/\lambda]\,x_i \end{Bmatrix}$$

Even with the assumptions of constant price and income elasticities and income shares, these equations cannot be solved directly for the administrative cost adjusted tax rates because the right hand sides include various β's that are functions of the tax rates. To isolate the tax rates, note that the net social marginal valuations of additional income to individuals A and B, taking into account the additional net tax revenues, net of administrative costs, generated from purchases of taxed goods financed with the additional income, are

$$\beta^A = \gamma^A\alpha^A + \lambda(1-c_t)t\left(\frac{\partial x_N^A}{\partial M^A}+\frac{\partial x_S^A}{\partial M^A}\right)$$

$$\beta^B = \gamma^B\alpha^B + \lambda(1-c_t)t\left(\frac{\partial x_N^B}{\partial M^B}+\frac{\partial x_S^B}{\partial M^B}\right)+\lambda(1-c_\tau)\tau\frac{\partial x_I^B}{\partial M^B}$$

or, defining η_i^h and ω_i^h as the income elasticities and income shares (inclusive of sales taxes paid) of individual h for good i and dividing by λ , yields

$$\beta^A/\lambda = \gamma^A \alpha^A/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^A \eta_N^A + \omega_S^A \eta_S^A\right)$$

$$\beta^B/\lambda = \gamma^B \alpha^B/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^B \eta_N^B + \omega_S^B \eta_S^B\right) + (1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B \eta_I^B) ,$$

These expressions are used in the definitions of β_N/λ, β_S/λ and β_I/λ, which are the social marginal utilities of revenues obtained from taxing the three taxable goods, expressed in revenue dollars, or

$$\beta_N/\lambda = \frac{N^A x_N^A}{x_N}\beta^A/\lambda + \frac{N^B x_N^B}{x_N}\beta^B/\lambda$$

$$\beta_S/\lambda = \frac{N^A x_S^A}{x_S}\beta^A/\lambda + \frac{N^B x_S^B}{x_S}\beta^B/\lambda$$

$$\beta_I/\lambda = \frac{N^B x_I^B}{x_I}\beta^B/\lambda = \beta^B/\lambda \ .$$

Substituting yields

$$\beta_N/\lambda = \frac{N^A x_N^A}{x_N}\left[\gamma^A \alpha^A/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^A \eta_N^A + \omega_S^A \eta_S^A\right)\right]$$
$$+ \frac{N^B x_N^B}{x_N}\left[\gamma^B \alpha^B/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^B \eta_N^B + \omega_S^B \eta_S^B\right) + (1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B \eta_I^B)\right]$$

$$\beta_S/\lambda = \frac{N^A x_S^A}{x_S}\left[\gamma^A \alpha^A/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^A \eta_N^A + \omega_S^A \eta_S^A\right)\right]$$
$$+ \frac{N^B x_S^B}{x_S}\left[\gamma^B \alpha^B/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^B \eta_N^B + \omega_S^B \eta_S^B\right) + (1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B \eta_I^B)\right]$$

$$\beta_I/\lambda = \frac{N^B x_I^B}{x_I}\beta^B/\lambda = \frac{N^B x_I^B}{x_I}\left[\gamma^B \alpha^B/\lambda + (1-c_t)\frac{t}{1+t}\left(\omega_N^B \eta_N^B + \omega_S^B \eta_S^B\right) + (1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B \eta_I^B)\right] .$$

As noted above, these expressions are functions of the two tax rates.

Substituting into the two first order conditions above yields

$$\begin{Bmatrix} N^A[x_N^A(\varepsilon_{Nw}^A+\varepsilon_{NE}^A)+x_S^A(\varepsilon_{Sw}^A+\varepsilon_{SE}^A)] \\ +N^B[x_N^B(\varepsilon_{Nw}^B+\varepsilon_{NE}^B+\varepsilon_{NI}^B)+x_S^B(\varepsilon_{Sw}^B+\varepsilon_{SE}^B+\varepsilon_{SI}^B)] & -N^B(x_N^B\varepsilon_{NI}^B+x_S^B\varepsilon_{SI}^B) \\ N^Bx_I^B(\varepsilon_{Iw}^B+\varepsilon_{IE}^B+\varepsilon_{II}^B) & -N^Bx_I^B\varepsilon_{II}^B \end{Bmatrix}\begin{Bmatrix}(1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau)\end{Bmatrix}$$

$$=\begin{Bmatrix} (1-c_t)x_N-N^Ax_N^A\left[\gamma^A\alpha^A/\lambda+(1-c_t)\frac{t}{1+t}\left(\omega_N^A\eta_N^A+\omega_S^A\eta_S^A\right)\right] \\ -N^Bx_N^B\left[\gamma^B\alpha^B/\lambda+(1-c_t)\frac{t}{1+t}\left(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B\right)+(1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B\eta_I^B)\right] \\ +(1-c_t)x_S-N^Ax_S^A\left[\gamma^A\alpha^A/\lambda+(1-c_t)\frac{t}{1+t}\left(\omega_N^A\eta_N^A+\omega_S^A\eta_S^A\right)\right] \\ -N^Bx_S^B\left[\gamma^B\alpha^B/\lambda+(1-c_t)\frac{t}{1+t}\left(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B\right)+(1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B\eta_I^B)\right] \\ (1-c_\tau)x_I-N^Bx_I^B\left[\gamma^B\alpha^B/\lambda+(1-c_t)\frac{t}{1+t}(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B)+(1-c_\tau)\frac{\tau}{1+\tau}(\omega_I^B\eta_I^B)\right] \end{Bmatrix}$$

Rearranging yields

$$\begin{Bmatrix} N^A[x_N^A(\varepsilon_{Nw}^A+\varepsilon_{NE}^A)+x_S^A(\varepsilon_{Sw}^A+\varepsilon_{SE}^A)] \\ +N^B[x_N^B(\varepsilon_{Nw}^B+\varepsilon_{NE}^B+\varepsilon_{NI}^B)+x_S^B(\varepsilon_{Sw}^B+\varepsilon_{SE}^B+\varepsilon_{SI}^B)] & -N^B(x_N^B\varepsilon_{NI}^B+x_S^B\varepsilon_{SI}^B) \\ +N^Ax_N^A(\omega_N^A\eta_N^A+\omega_S^A\eta_S^A)+N^Bx_N^B(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B) & +N^Bx_N^B(\omega_I^B\eta_I^B) \\ +N^Ax_S^A(\omega_N^A\eta_N^A+\omega_S^A\eta_S^A)+N^Bx_S^B(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B) & +N^Bx_S^B(\omega_I^B\eta_I^B) \\ N^Bx_I^B(\varepsilon_{Iw}^B+\varepsilon_{IE}^B+\varepsilon_{II}^B)+N^Bx_I^B(\omega_N^B\eta_N^B+\omega_S^B\eta_S^B) & -N^Bx_I^B\varepsilon_{II}^B+N^Bx_I^B(\omega_I^B\eta_I^B) \end{Bmatrix}\begin{Bmatrix}(1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau)\end{Bmatrix}$$

$$=\begin{Bmatrix} (1-c_t)x_N-N^Ax_N^A(\gamma^A\alpha^A/\lambda)-N^Bx_N^B(\gamma^B\alpha^B/\lambda)+(1-c_t)x_S-N^Ax_S^A(\gamma^A\alpha^A/\lambda)-N^Bx_S^B(\gamma^B\alpha^B/\lambda) \\ (1-c_\tau)x_I-N^Bx_I^B(\gamma^B\alpha^B/\lambda) \end{Bmatrix}$$

Denoting the matrix elements as follows

$$\begin{Bmatrix} a & b \\ c & d \end{Bmatrix}\begin{Bmatrix}(1-c_t)t/(1+t) \\ (1-c_\tau)\tau/(1+\tau)\end{Bmatrix}=\begin{Bmatrix} e \\ f \end{Bmatrix}$$

the ratio of the administrative cost adjusted tax rates can be calculated as

$$\frac{(1-c_t)}{(1-c_\tau)}\frac{[t/(1+t)]}{[\tau/(1+\tau)]}=\frac{ed-bf}{af-ec}$$

where the expression on the right hand side has no tax rate terms. This condition yields the expression for the tax rate on Internet goods as a function of the tax rate on other taxed goods, or

$$\frac{\tau}{1+\tau}=\left(\frac{af-ec}{ed-bf}\right)\left(\frac{1-c_t}{1-c_\tau}\right)\frac{t}{1+t} \quad .$$

To solve for τ,

$$\frac{\tau}{1+\tau}=\left(\frac{af-ec}{ed-bf}\right)\left(\frac{1-c_t}{1-c_\tau}\right)\frac{t}{1+t} \quad =z\frac{t}{1+t}; \quad z=\left(\frac{af-ec}{ed-bf}\right)\left(\frac{1-c_t}{1-c_\tau}\right)$$

$$\tau=z\frac{t}{1+t}(1+\tau)$$

$$\tau\left(1-z\frac{t}{1+t}\right)=z\frac{t}{1+t}$$

$$\tau=\frac{z\frac{t}{1+t}}{1-z\frac{t}{1+t}}=\frac{zt}{1+t-zt}=\frac{zt}{1+(1-z)t}$$

Thus, if B_t and B_τ are the bases of the two taxes, the revenue constraint – neglecting changes in the two bases – can be solved for the two tax rates t and τ:

$$R + C_t t B_t + C_\tau \tau B_\tau = tB_t + \tau B_\tau$$

$$R = t(1 - C_t)B_t + \frac{zt}{1 + (1 - z)t}(1 - C_\tau)B_\tau$$

$$R[1 + (1 - z)t] = [1 + (1 - z)t]t(1 - C_t)B_t + zt(1 - C_\tau)B_\tau$$

$$[(1 - z)(1 - C_t)B_t]t^2 + [(1 - C_t)B_t + z(1 - C_\tau)B_\tau - R(1 - z)]t - R = 0$$

$$t = \frac{-[(1 - C_t)B_t + z(1 - C_\tau)B_\tau - R(1 - z)]}{2[(1 - z)(1 - C_t)B_t]}$$

$$\pm \frac{\sqrt{[(1 - C_t)B_t + z(1 - C_\tau)B_\tau - R(1 - z)]^2 + 4[(1 - z)(1 - C_t)B_t]R}}{2[(1 - z)(1 - C_t)B_t]}$$

where

$$\tau = \frac{zt}{1 + (1 - z)t}.$$